Matrix mult is not commutative

2.1 cont: Note $\boxed{AB \neq BA}$

$$\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} (1)(1)+1(1) & (1)(1)+(1)(1) \\ (-1)(1)+(-1)(1) & (-1)(1)+(-1)(1) \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix}$$

$$\neq$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

It is also possible that $AB = AC$, but $B \neq C$.

In particular it is possible for $AB = 0$, but $A \neq 0$ AND $B \neq 0$

Defn: If A is a square $(n \times n)$ matrix, $A^0 = I$, $A^1 = A$, $A^k = AA...A$.

$2^0 = 1$

Ex: $$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} (1)(1)+(2)(3) & (1)(2)+(2)(4) \\ 3(1)+4(3) & 3(2)+(4)(4) \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

The transpose of the $m \times n$ matrix $A = A^T = (a_{ji})$.

Ex: $$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Transpose Properties:

obvious → a.) $(A^T)^T = A$

b.) $(A + B)^T = A^T + B^T$

c.) $(kA)^T = kA^T$

d.) $(AB)^T = B^T A^T$

$A = (a_{ij})$

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

matrix mult makes sense

2.2:

Defn: A is invertible if there exists a matrix B such that $AB = BA = I$, and B is called the inverse of A. If the inverse of A does not exist, then A is said to be singular.

Note that if A is invertible, then A is a square matrix.

$$\underset{m \times k}{A} \ \underset{k \times n}{B} = \underset{k \times n}{B} \ \underset{m \times k}{A}$$

$$\Rightarrow n = m$$

Thm: If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then A is invertible if and only if $ad - bc \neq 0$, in which case

2X2

$$\boxed{A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}$$

ch 3
ad-bc
= determin

Ex: The inverse of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is $\begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$

since

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 4/-2 & -2/-2 \\ -3/-2 & 1/-2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3/2 & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \blacksquare$$

$ad - bc = (1)(4) - 3(2) = 4 - 6 = -2$

Find the inverse of $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 5 & 6 \\ 6 & 7 & 9 \end{bmatrix}$.

Long method:
$$\begin{bmatrix} 2 & 3 & 4 \\ 4 & 5 & 6 \\ 6 & 7 & 9 \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} =$$

$$\begin{bmatrix} 2x_{11} + 3x_{21} + 4x_{31} & 2x_{12} + 3x_{22} + 4x_{32} & 2x_{13} + 3x_{23} + 4x_{33} \\ 4x_{11} + 5x_{21} + 6x_{31} & 4x_{12} + 5x_{22} + 6x_{32} & 4x_{13} + 5x_{23} + fx_{33} \\ \boxed{6x_{11} + 7x_{21} + 9x_{31}} & 6x_{12} + 7x_{22} + 9x_{32} & 6x_{13} + 7x_{23} + 9x_{33} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \boxed{0} & 0 & 1 \end{bmatrix}.$$

So solve,

$$\left[\begin{array}{ccc|c} 2 & 3 & 4 & 1 \\ 4 & 5 & 6 & 0 \\ 6 & 7 & 9 & 0 \end{array}\right] \text{ for } x_{11}, x_{21}, x_{31}. \Rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ is in span of col of } A$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 4 & 0 \\ 4 & 5 & 6 & 1 \\ 6 & 7 & 9 & 0 \end{array}\right] \text{ for } x_{12}, x_{22}, x_{32}. \Rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad ''$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 4 & 0 \\ 4 & 5 & 6 & 0 \\ 6 & 7 & 9 & 1 \end{array}\right] \text{ for } x_{13}, x_{23}, x_{33}. \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad ''$$

$\Downarrow$

span $= \mathbb{R}^3$

if A^{-1} exists

Or shorter method, solve

$$\left[\begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 0 & 0 \\ 4 & 5 & 6 & 0 & 1 & 0 \\ 6 & 7 & 9 & 0 & 0 & 1 \end{array}\right]$$

Solving 9 systems of equations

A^{-1} exists $\Longleftrightarrow$ $[A \mid I]$ has a sol'n

$$\left[\overbrace{\begin{matrix}2&3&4\\4&5&6\\6&7&9\end{matrix}}^{A}\;\middle|\;\overbrace{\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}}^{I}\right]$$

$\downarrow (R_2 - 2R_1 \to R_2, R_3 - 3R_1 \to R_3)$

$$\left[\begin{array}{ccc|ccc}2&3&4&1&0&0\\0&-1&-2&-2&1&0\\0&-2&-3&-3&0&1\end{array}\right]$$

$\downarrow (-R_2 \to R_2)$

$$\left[\begin{array}{ccc|ccc}2&3&4&1&0&0\\0&1&2&2&-1&0\\0&-2&-3&-3&0&1\end{array}\right]$$

$\downarrow (R_3 + 2R_2 \to R_3)$

$$\left[\begin{array}{ccc|ccc}2&3&4&1&0&0\\0&1&2&2&-1&0\\0&0&1&1&-2&1\end{array}\right]$$

$\downarrow (R_1 - 4R_3 \to R_1, R_2 - 2R_3 \to R_2)$

$$\begin{bmatrix}2&3&0&-3&8&-4\\0&1&0&0&3&-2\\0&0&1&1&-2&1\end{bmatrix}$$

$\downarrow (R_1 - 3R_2 \to R_1)$

$$\begin{bmatrix}2&0&0&-3&-1&2\\0&1&0&0&3&-2\\0&0&1&1&-2&1\end{bmatrix} \xrightarrow[(\frac{1}{2}R_1 \to R_1)]{} \left[\underbrace{\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}}_{I}\;\middle|\;\underbrace{\begin{matrix}-\frac{3}{2}&-\frac{1}{2}&1\\0&3&-2\\1&-2&1\end{matrix}}_{A^{-1}}\right]$$

A is invertible $\iff$ $A \sim I$, A square

$$\text{Thus } \left[\begin{array}{ccc|c} 2 & 3 & 4 & 1 \\ 4 & 5 & 6 & 0 \\ 6 & 7 & 9 & 0 \end{array}\right] \text{ is row equivalent to } \begin{array}{c} \begin{array}{ccc} x_{11} & x_{21} & x_{31} \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{3}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right] \end{array},$$

so $(x_{11}, x_{21}, x_{31}) = (-\frac{3}{2}, 0, 1)$.

$$\left[\begin{array}{ccc|c} 2 & 3 & 4 & 0 \\ 4 & 5 & 6 & 1 \\ 6 & 7 & 9 & 0 \end{array}\right] \text{ is row equivalent to } \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right],$$

so $(x_{12}, x_{22}, x_{32}) = (\frac{-1}{2}, 3, -2)$.

$$\left[\begin{array}{ccc|c} 2 & 3 & 4 & 0 \\ 4 & 5 & 6 & 0 \\ 6 & 7 & 9 & 1 \end{array}\right] \text{ is row equivalent to } \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right],$$

so $(x_{13}, x_{23}, x_{33}) = (1, -2, 1)$.

When is A invertible??

Shortest method:

Note that if $[A|I]$ is row equivalent to $[I|B]$, then $B = A^{-1}$.

$$\text{Thus the inverse of } \begin{bmatrix} 2 & 3 & 4 \\ 4 & 5 & 6 \\ 6 & 7 & 9 \end{bmatrix} \text{ is } \begin{bmatrix} -\frac{3}{2} & -\frac{1}{2} & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\text{Check answer: } \begin{bmatrix} 2 & 3 & 4 \\ 4 & 5 & 6 \\ 6 & 7 & 9 \end{bmatrix} \begin{bmatrix} -\frac{3}{2} & -\frac{1}{2} & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solving system of 9 equations for 9 unknowns

Claim A is invertible

$\iff [A \mid I] \sim [I \mid A^{-1}]$

Case A 3×3 matrix

A has 3 pivots (assuming A invertible)

If A is $\boxed{\text{invertible}}$

there is a soln to

$$[\underbrace{A}_{9 \text{ variables}} \mid \underbrace{I}_{3 \text{ columns}}]$$

$\left[A \,\middle|\, \begin{matrix}1\\0\\0\end{matrix}\right]$ soln $\Rightarrow$ $\begin{bmatrix}1\\0\\0\end{bmatrix}$ is in span of col of A

$\left[A \,\middle|\, \begin{matrix}0\\1\\0\end{matrix}\right]$ " $\Rightarrow$ $\begin{bmatrix}0\\1\\0\end{bmatrix}$ "

$\left[A \,\middle|\, \begin{matrix}0\\0\\1\end{matrix}\right]$ " $\Rightarrow$ $\begin{bmatrix}0\\0\\1\end{bmatrix}$ "

$\begin{bmatrix}1\\0\\0\end{bmatrix}$ $\begin{bmatrix}0\\1\\0\end{bmatrix}$ $\begin{bmatrix}0\\0\\1\end{bmatrix}$ are all in span of col of A

$\Rightarrow$ all ~~the~~ LINEAR COMBINATIONS are in span of col of A

$$\text{span}\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\} = \mathbb{R}^3$$

span of col of A $= \mathbb{R}^3$

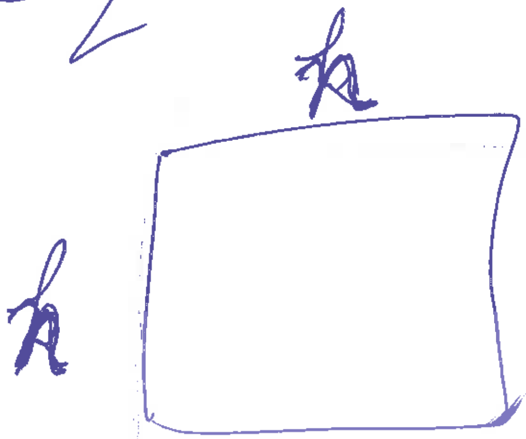
A is invertable $\Leftrightarrow$ span of col of A $= \mathbb{R}^3$

$\Leftrightarrow$ $[A]$ has 3 pivots

A 3×3 no free variables

(Solution is unique)

Square



lin indep $\iff$ 0 free variables $\iff$ k pivots $\iff$ span $= \mathbb{R}^k$

$\boxed{\text{square}}$

A^{-1} exists $\Rightarrow$ A square $n \times n$

A^{-1} exists $\Leftarrow$ span of col of $A = \mathbb{R}^n$

$\Leftrightarrow$ $[\underbrace{A}_{\text{no row of all 0's for coef matrix}} \mid I] \sim [I \mid A^{-1}]$

$\Leftrightarrow$ n pivots

$\Leftrightarrow$ no free variables

$\Leftrightarrow$ $A \sim I$

$[A \mid I]$ has a soln

$\Longleftrightarrow$ the columns of A span $\mathbb{R}^n$

for A $\cancel{a}$ $n \times n$ matrix

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix} \cdots \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

is in the span of A

12:30

WHEN DOES A^{-1} EXIST? $A \sim I$

$$[A \mid I] \sim [I \mid A^{-1}]$$

$$\cancel{A^{-1} A} x = A^{-1} b$$

unique soln

$$x = A^{-1} b$$

$\Leftrightarrow$ A is invertible

Solve $2x + 3y + 4z = 0$
$4x + 5y + 6z = 0$
$6x + 7y + 9z = 0$

$$\left[A \,\middle|\, \begin{matrix}0\\0\\0\end{matrix}\right] \sim \left[I \,\middle|\, \begin{matrix}0\\0\\0\end{matrix}\right] = \left[\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix} \,\middle|\, \begin{matrix}0\\0\\0\end{matrix}\right] \Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

By Monday's lecture $\Rightarrow$ unique sol'n

Solve $2x + 3y + 4z = 0$
$4x + 5y + 6z = 2$
$6x + 7y + 9z = 1$

Don't do long method if A^{-1} Known

Very Long Method when inverse is Known

$$\left[\begin{matrix}2&3&4\\4&5&6\\6&7&9\end{matrix} \,\middle|\, \begin{matrix}0\\2\\1\end{matrix}\right] \sim \left[\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix} \,\middle|\, \begin{matrix}\emptyset\\\emptyset\\\emptyset\end{matrix}\right]$$

Nice Short Method if A^{-1} exists & is Known

$$\cancel{A^{-1} \begin{bmatrix}2&3&4\\4&5&6\\6&7&9\end{bmatrix}} \begin{bmatrix}x\\y\\z\end{bmatrix} = A^{-1} \begin{bmatrix}0\\2\\1\end{bmatrix}$$

$$\cancel{A^{-1} A}\, \vec{x} = A^{-1} \vec{b}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3/2 & -1/2 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 - 1 + 1 \\ 0 + 6 - 2 \\ 0 - 4 + 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -3 \end{bmatrix}$$

Warning!!! $AB \neq BA$

$$A^{-1}(Ax) = A^{-1}(b)$$

Do identical thing to both sides

Multiply both sides by A^{-1} on the left

Find the inverse if it exists & use it to solve the following eqns

$$A = \begin{bmatrix} 2 & 1 \\ 8 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -3/2 & 1/2 \\ 4 & -1 \end{bmatrix}$$ see scratch

---

Solve $\begin{bmatrix} 2 & 1 \\ 8 & 3 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 \\ 8 & 3 \end{bmatrix} \vec{x} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 8 \end{bmatrix} x_1 + \begin{bmatrix} 1 \\ 3 \end{bmatrix} x_2 = \begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

Scratch

$$\begin{bmatrix} 8 & 2 \\ & \end{bmatrix}\left[\begin{array}{c|c} 2 & 1 \\ 8 & 3 \end{array}\right] = \begin{bmatrix} 0 & \\ & \end{bmatrix}$$

$$\begin{bmatrix} 3/-2 & -1/-2 \\ -8/-2 & 2/-2 \end{bmatrix}\left[\begin{array}{c|c} 2 & 1 \\ 8 & 3 \end{array}\right] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -3/2 & 1/2 \\ 4 & -1 \end{bmatrix}$$

check:

$$AA^{-1} = \begin{bmatrix} 2 & 1 \\ 8 & 3 \end{bmatrix}\begin{bmatrix} -3/2 & 1/2 \\ 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3+4 & 1-1 \\ -12+12 & 4-3 \end{bmatrix} = I_{2\times 2}$$

✓

side question

If $A^{-1} = B = (b_{ij})$

Find $b_{21} = 4$

row column

Solve

$$\begin{bmatrix} -3/2 & 1/2 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 8 & 3 \end{bmatrix}\vec{x} = \begin{bmatrix} -3/2 & 1/2 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} -3/2 - 3/2 \\ 4 + 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$$

Solve

$$\begin{bmatrix} -3/2 & 1/2 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 8 & 3 \end{bmatrix}\vec{x} = \begin{bmatrix} -3/2 & 1/2 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

(Always multiply on same side)
eg in this example, on the left

$$\vec{x} = \begin{bmatrix} -6 + 5/2 \\ 16 - 5 \end{bmatrix} = \begin{bmatrix} -7/2 \\ 11 \end{bmatrix}$$

Solve $\begin{bmatrix}2\\8\end{bmatrix}x_1 + \begin{bmatrix}1\\3\end{bmatrix}x_2 = \begin{bmatrix}10\\-10\end{bmatrix}$

$$\cancel{A^{-1}\begin{bmatrix}2&1\\8&3\end{bmatrix}}\begin{bmatrix}x_1\\x_2\end{bmatrix} = A^{-1}\begin{bmatrix}10\\-10\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}-3/2 & 1/2\\4 & -1\end{bmatrix}\begin{bmatrix}10\\-10\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}-10\\50\end{bmatrix}$$

$x_1 = -10 \qquad x_2 = 50$

Find A^{-1} if it exists & use to solve following eq'ns

where $A = \begin{bmatrix} 5 & 2 \\ 4 & 2 \end{bmatrix}$

① Find A^{-1}

$$A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 5/2 \end{bmatrix}$$

Solve $\begin{bmatrix} 5 & 2 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

$$5x + 2y = 2$$
$$4x + 2y = 1$$

$$\begin{bmatrix} 5 \\ 4 \end{bmatrix} x + \begin{bmatrix} 2 \\ 2 \end{bmatrix} y = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

Scratch

$$\begin{array}{cc} A^{-1} & A \\ \begin{bmatrix} 2/2 & -2/2 \\ -4/2 & 5/2 \end{bmatrix} & \begin{bmatrix} 5 & 2 \\ 4 & 2 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(2)(5) - 2(4) = 10 - 8 = 2$$

$$A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 5/2 \end{bmatrix}$$

Check: $A\ A^{-1}$

$$\begin{bmatrix} 5 & 2 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -2 & 5/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

✓

$$\underline{AA^{-1} = I = A^{-1}A}$$

side problem

Suppose $A^{-1} = B = (b_{ij})$ $= \begin{bmatrix} 1 & -1 \\ -2 & 5/2 \end{bmatrix}$

or $A = (a_{ij})$ $B = (b_{ij})$

$AB = I = BA$

then $b_{21} = \underline{-2}$

2nd row, 1rst column

Solve $\cancel{\begin{bmatrix} 1 & -1 \\ -2 & 5/2 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 4 & 2 \end{bmatrix}}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 5/2 \end{bmatrix}\begin{bmatrix} 3 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$$

Solve
$$\begin{aligned} 5x + 2y &= 2 \\ 4x + 2y &= 1 \end{aligned}$$

$$\cancel{A^{-1}\begin{bmatrix} 5 & 2 \\ 4 & 2 \end{bmatrix}}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 5/2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 - 1 \\ -4 + 5/2 \end{bmatrix} = \begin{bmatrix} 1 \\ -3/2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -3/2 \end{bmatrix}$$

Solve $\begin{bmatrix}5\\4\end{bmatrix}x + \begin{bmatrix}2\\2\end{bmatrix}y = \begin{bmatrix}10\\10\end{bmatrix}$

$$\cancel{A^{-1}\begin{bmatrix}5 & 2\\4 & 2\end{bmatrix}}\begin{bmatrix}x\\ \hline y\end{bmatrix} = \begin{bmatrix}1 & -1\\-2 & 5/2\end{bmatrix}\begin{bmatrix}10\\10\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}10 & -10\\-20 & +\frac{50}{2}\end{bmatrix} = \begin{bmatrix}0\\5\end{bmatrix}$$

$x = 0 \quad y = 5$

$$\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\5\end{bmatrix}$$

Solve $\underbrace{\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}} \vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

span = 1-dim line

inverse does not exist

so we will either have no soln or ∞ # of solns

Can't solve using A^{-1}

SIDE NOTE (ch 3) $(1)(6) - (3)(2) = 0$

Thm 8′: If A is a SQUARE $n \times n$ matrix, then the following are equivalent.

a.) A is invertible.

b.) The row-reduced echelon form of A is I_n, the identity matrix.

c.) An echelon form of A has n leading entries
[I.e., every column of an echelon form of A is a leading entry column – no free variables]. (A square => A has leading entry in every column if and only if A has leading entry in every row). no row of all 0's

d.) The column vectors of A are linearly independent.

e.) $Ax = 0$ has only the trivial solution.

f.) $Ax = b$ has at most one sol'n for any b.

g.) $Ax = b$ has a unique sol'n for any b. SQUARE

h.) $Ax = b$ is consistent for every $n \times 1$ matrix b.

i.) $Ax = b$ has at least one sol'n for any b.

j.) The column vectors of A span R^n.
[every vector in R^n can be written as a linear combination of the columns of A].

k.) There is a square matrix C such that $CA = I$.

l.) There is a square matrix D such that $AD = I$.

m.) A^T is invertible.

n.) A is expressible as a product of elementary matrices.