12:30

1.5

Solve the following systems of equations:

REF

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

REF
$$\begin{array}{c} \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right] \\ \begin{array}{cc} X_1 & X_2 \end{array} \end{array}$$

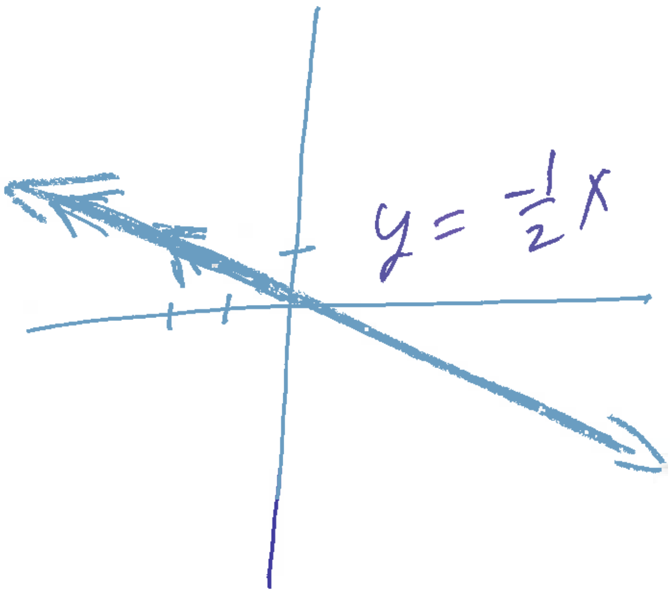
$$X_1 = -2X_2$$
$$X_2 = X_2$$

$$\Rightarrow \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -2X_2 \\ X_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} X_2$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} X_2$$

Sol'n $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} x_2$

$= \begin{bmatrix} -2 \\ 1 \end{bmatrix} y$



$= \begin{bmatrix} -2 \\ 1 \end{bmatrix} s$

$= \begin{bmatrix} -2 \\ 1 \end{bmatrix} a$

Sol'n set

= line $\begin{bmatrix} -2 \\ 1 \end{bmatrix} t = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $= \begin{bmatrix} -4 \\ 2 \end{bmatrix} s$

$y = \frac{-1}{2} x$ ← line thru origin $= \begin{bmatrix} 2 \\ -1 \end{bmatrix} u$

$= \begin{bmatrix} 6 \\ -3 \end{bmatrix} t$

all represent the same line = solution set

all are correct answer

$$\begin{array}{cc} & \text{fv} \\ \left[\begin{matrix} 1 \\ 0 \end{matrix}\right. & \left.\boxed{\begin{matrix} 2 \\ 0 \end{matrix}}\right] \end{array} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \longrightarrow \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

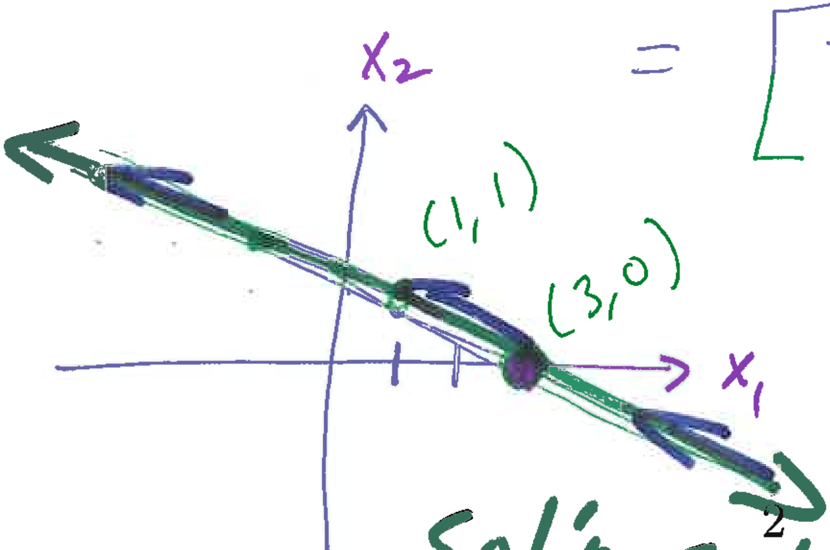
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x_2$$

$$x_1 = -2x_2 + 3$$
$$x_2 = x_2$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2x_2 + 3 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2x_2 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} x_2 + \begin{bmatrix} 3 \\ 0 \end{bmatrix}}$$

$$= \begin{bmatrix} -4 \\ 2 \end{bmatrix} u + \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$



$$= \underbrace{\begin{bmatrix} -1 \\ 1/2 \end{bmatrix}}_{\text{any vector giving direction}} t + \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\text{any pt on line}}$$

Sol'n set = line thru pt $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ w/ direction $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$$\begin{bmatrix}-2\\1\end{bmatrix}s + \begin{bmatrix}-2\\1\end{bmatrix}$$

not be a sol'n

to $\begin{bmatrix}1 & 2\\0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}3\\0\end{bmatrix}$

Pf: Let $s = 0$

$$\begin{bmatrix}1 & 2\\0 & 0\end{bmatrix}\begin{bmatrix}-2\\1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$\Rightarrow \begin{bmatrix}-2\\1\end{bmatrix}s + \begin{bmatrix}-2\\1\end{bmatrix}$

is a <u>non</u>-simplified sol'n to homog

<u>NOT-SIMPLIFIED</u> = <u>NOT</u> CORRECT

Non-homog sol'n

= homog + a point on line representing non homog sol'n

$$\begin{bmatrix} -2 \\ 1 \end{bmatrix} s + \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 1 \end{bmatrix} s + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 1 \end{bmatrix} s + \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$\boxed{\text{homog}}$ | Solve = REF | $\boxed{\text{non-homog}}$

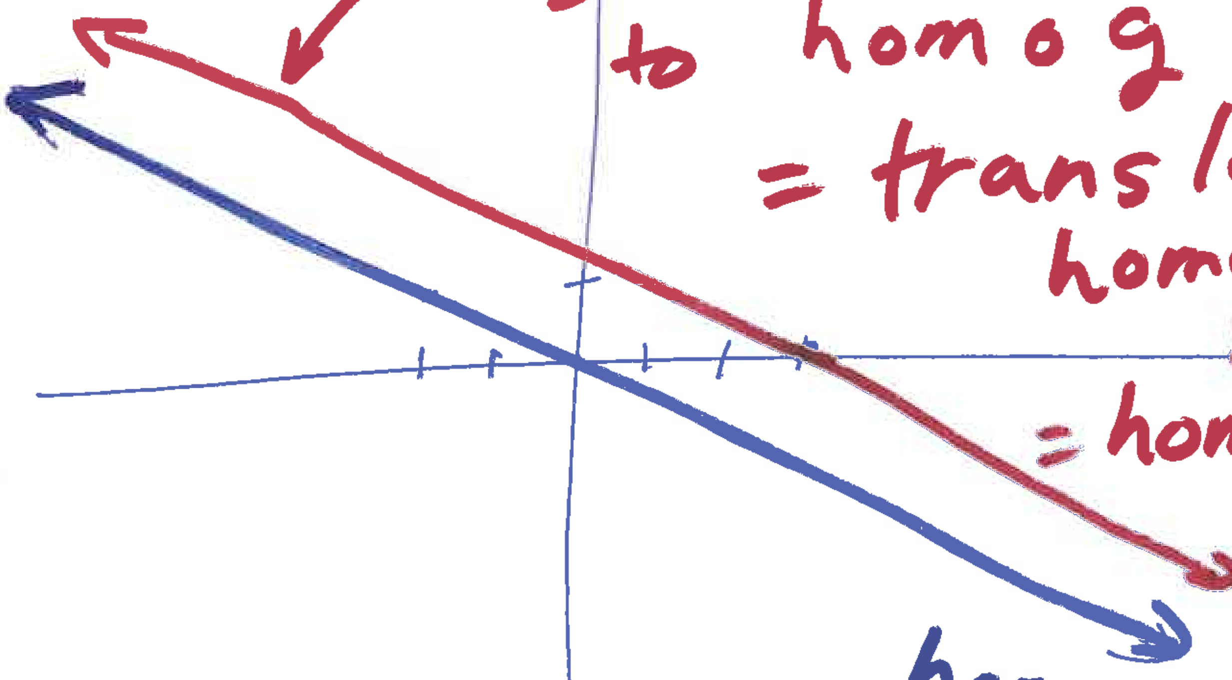
$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad\qquad \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -2 \\ 1 \end{bmatrix} x_2}_{\text{homog}} \qquad\qquad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -2 \\ 1 \end{bmatrix} x_2}_{\text{homog}} + \underbrace{\begin{bmatrix} 3 \\ 0 \end{bmatrix}}_{\text{a non-homo sol'n}}$$

Note: non-homog sol'n is parallel to homog

= translated homog sol'n

= homog + $[1]$



homog sol'n

= line thru origin

10:30

REF

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_1 + 2x_2 + 3x_3 = 0$$

$$x_1 = -2x_2 - 3x_3$$

$$x_2 = x_2$$

$$x_3 = x_3$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_2 - 3x_3 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_2 \\ x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} -3x_3 \\ 0 \\ x_3 \end{bmatrix}$$

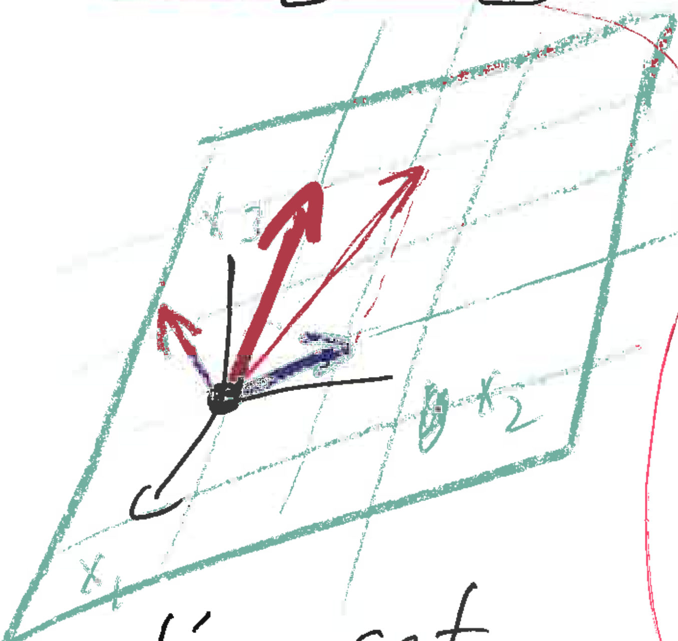
$$= \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} x_3$$

sol'n set = 2d plane

soln set to homog eqn

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \vec{x} = \vec{0}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} x_3$$



$$= \begin{bmatrix} -5 \\ 1 \\ 1 \end{bmatrix} s + \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} t$$

both correct sol'ns

Both describe same plane

$$= \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -6 \\ 0 \\ 2 \end{bmatrix} t$$

sol'n set = plane thru $\vec{0}$ (since homog eqn)

Note we can determine if a vector lies in hyperplane describing soln set of $Ax = 0$, by plugging it in.

Does it satisfy the equation $Ax = 0$?

If so, it is the soln set

Check $\begin{bmatrix} -5 \\ 1 \\ 1 \end{bmatrix}$ satisfies $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -5 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -5+2+3 \\ 0+0+0 \\ 0+0+0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \checkmark$$

$$x_1 + 2x_2 + 3x_3 = 0$$
$$0 = 0$$
$$0 = 0$$

$$-5 + 2(1) + 3(1) = 0 \checkmark$$

If $\vec{x} = s\vec{v} + t\vec{w}$ describes the sol'n set of $A\vec{x} = \vec{0}$

$A\vec{v} = \vec{0}$ & $A\vec{w} = \vec{0}$

Answers must be simplified

Non-simplified answer:

$$\vec{x} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} t + \begin{bmatrix} -5 \\ 1 \\ 1 \end{bmatrix} u$$

USE REF

Describes same plane but only 2 vectors needed to describe 2-d plane

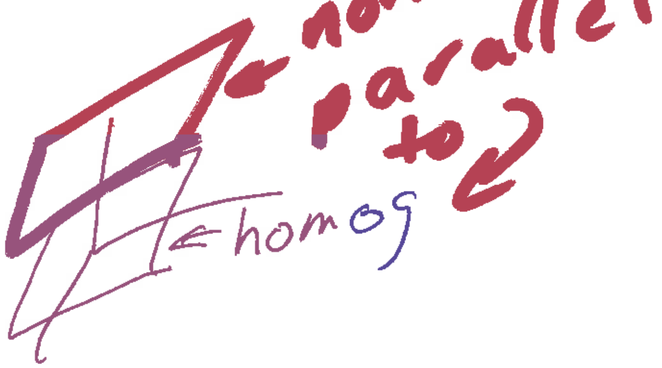
not needed to describe 2-d plane

10:30

REF

$$\begin{bmatrix}1 & 2 & 3\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}4\\ 0\\ 0\end{bmatrix} \to \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{array}\right] \end{array}$$

$$\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}-2x_2-3x_3+4\\ x_2\\ x_3\end{bmatrix} = \underbrace{x_2\begin{bmatrix}-2\\ 1\\ 0\end{bmatrix} + x_3\begin{bmatrix}-3\\ 0\\ 1\end{bmatrix}}_{\text{homog}} + \underbrace{\begin{bmatrix}4\\ 0\\ 0\end{bmatrix}}_{\text{a non homog sol'n}}$$



Note

Let $x_2 = 1$, $x_3 = 1$

$$\underbrace{\begin{bmatrix}-4\\2\\0\end{bmatrix}s + \begin{bmatrix}-5\\1\\1\end{bmatrix}t}_{\text{homog}} + \underbrace{\begin{bmatrix}-1\\1\\1\end{bmatrix}}_{\text{+ A sol'n to to non homog}}$$

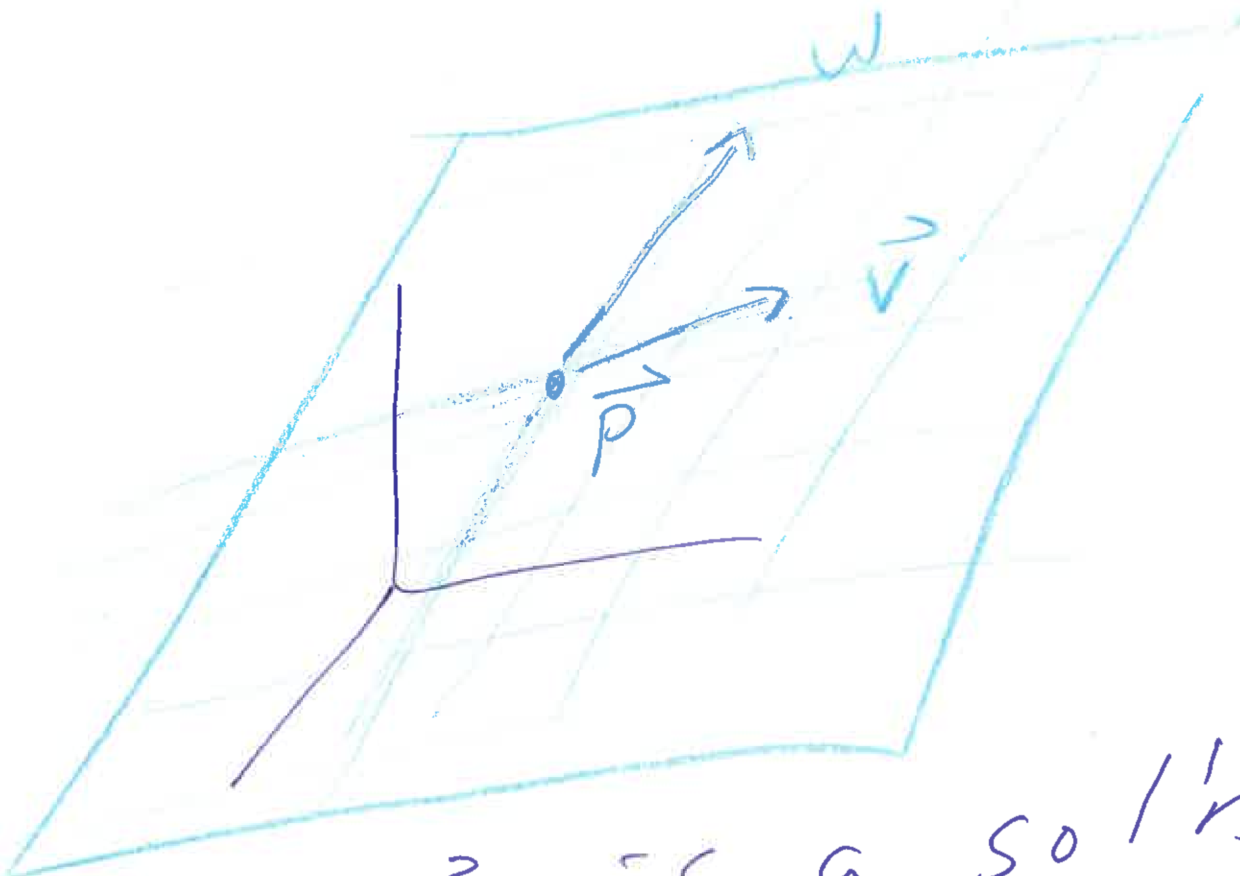
also describes plane

"   correct answer

$$\begin{bmatrix}1&2&3\\0&0&0\\0&0&0\end{bmatrix}\underbrace{\begin{bmatrix}-1\\1\\1\end{bmatrix}}_{\text{hence sol'n}} = \begin{bmatrix}-1+2+3\\0\\0\end{bmatrix} = \begin{bmatrix}4\\0\\0\end{bmatrix}$$

✓

$$\vec{x} = t\vec{v} + s\vec{w} + \vec{p}$$



Note $\vec{p}$ is a sol'n to non-homog

BUT $\vec{v}$ & $\vec{w}$ are NOT

$\vec{v}$ & $\vec{w}$ are sol'n to homog eq

REF

Solve: $B\mathbf{x} = \mathbf{0}$ where

$$B \sim \begin{array}{c} \left[\begin{array}{ccccc|c} \boxed{0} & 1 & 0 & \boxed{8} & 0 & 0 \\ 0 & 0 & 1 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \\ \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \end{array} \end{array}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} x_1 \\ -8x_4 \\ \cancel{-}+6x_4 \\ x_4 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -8x_4 \\ \cancel{-}+6x_4 \\ x_4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ -8 \\ \cancel{-}+6 \\ 1 \\ 0 \end{bmatrix} x_4$$

REF

Solve: $B\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ where $B \sim$ $\begin{array}{cccccc} \left[\begin{matrix} \boxed{0} & 1 & 0 & \boxed{8} & 0 \\ 0 & 0 & 1 & -6 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{matrix}\right. & \left|\begin{matrix} 1 \\ 2 \\ 3 \end{matrix}\right] \\ x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 & \end{array}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} x_1 \\ -8x_4 + 1 \\ -6x_4 + 2 \\ x_4 \\ 3 \end{bmatrix} = \begin{bmatrix} x_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -8x_4 \\ -6x_4 \\ x_4 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ -8 \\ -6 \\ 1 \\ 0 \end{bmatrix} x_4}_{\text{homog}} + \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}$$

homog + A soln to non homo

Solution to non-homog eqn

A NON-homogeneous system of LINEAR equations

a.) Exactly one solution.

b.) Infinite number of solutions

c.) No solutions

A system of equations is $A\mathbf{x} = \mathbf{b}$ is **homogeneous** if $\mathbf{b} = \mathbf{0}$.

A homogeneous system of LINEAR equations can have

a.) Exactly one solution ($\mathbf{x} = \mathbf{0}$)

b.) Infinite number of solutions (including, of course, $\mathbf{x} = \mathbf{0}$).

Solve the following systems of equations:

1

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

2

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$$

3

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$$

$$\begin{array}{c} \begin{array}{cccccc} & & & \text{eq'n 1} & \text{eq'n 2} & \text{eq'n 3} \end{array} \\ \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 0 & 2 \\ 4 & 5 & 6 & 0 & 3 & 5 \\ 7 & 8 & 9 & 0 & 0 & 8 \end{array}\right] \end{array}$$

$\downarrow R_2 - 4R_1 \to R_2, \quad R_3 - 7R_1 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 0 & 2 \\ 0 & -3 & -6 & 0 & 3 & -3 \\ 0 & -6 & -12 & -7 & 0 & -6 \end{array}\right]$$

$\downarrow R_3 - 2R_1 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 0 & 2 \\ 0 & -3 & -6 & 0 & 3 & -3 \\ 0 & 0 & 0 & 0 & -6 & 0 \end{array}\right]$$

no sol'n

EF

$\downarrow$ already know sol'n to system b.

$$\left[\begin{array}{ccc|cc} 1 & 2 & 3 & 0 & 2 \\ 0 & -3 & -6 & 0 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\downarrow -\frac{1}{3}R_2 \to R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 2 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow[R_1 - 2R_2 \to R_1]{} \left[\begin{array}{ccc|cc} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

REF

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & \\ 0 & 1 & 2 & \\ 0 & 0 & 0 & \end{array}\right]$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} x_3$$

homog

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$$

No sol'n

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & -6 \end{array}\right]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} \qquad \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} x_3 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$x_1 = +x_3$

$x_2 = -2x_3 + 1$

$x_3 = x_3$

Note that $A(\mathbf{x}+\mathbf{y}) = A\mathbf{x} + A\mathbf{y}$ and $A(c\mathbf{x}) = cA\mathbf{x}$

For example,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \right) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}(x_1 + y_1) & a_{12}(x_2 + y_2) \\ a_{21}(x_1 + y_1) & a_{22}(x_2 + y_2) \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}x_1 + a_{11}y_1 & a_{12}x_2 + a_{12}y_2 \\ a_{21}x_1 + a_{21}y_1 & a_{22}x_2 + a_{22}y_2 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}x_1 & a_{12}x_2 \\ a_{21}x_1 & a_{22}x_2 \end{bmatrix} + \begin{bmatrix} a_{11}y_1 & a_{12}y_2 \\ a_{21}y_1 & a_{22}y_2 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$2(x+y) = 2x+2y$

$A(x+y) = Ax+Ay$

$\sqrt{2+3} \neq \sqrt{2}+\sqrt{3}$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \left(c \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} cx_1 \\ cx_2 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}cx_1 & a_{12}cx_2 \\ a_{21}cx_1 & a_{22}cx_2 \end{bmatrix} = c \begin{bmatrix} a_{11}x_1 & a_{12}x_2 \\ a_{21}x_1 & a_{22}x_2 \end{bmatrix} = c \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Suppose $A\mathbf{u} = \mathbf{0}$, $A\mathbf{v} = \mathbf{0}$, and $A\mathbf{p} = \mathbf{b}$

$A\vec{u} = 0 \qquad A\vec{v} = 0$

$\vec{u}$ & $\vec{v}$ soln to homog eqn $A\vec{x} = 0$

Claim $s\vec{u} + t\vec{v}$ is also a soln to homog eqn $A\vec{x} = 0$

Plug it in to check:

$$A(s\vec{u} + t\vec{v})$$
$$= A(s\vec{u}) + A(t\vec{v})$$
$$= s(A\vec{u}) + t(A\vec{v})$$
$$= s\vec{0} + t\vec{0} = \overline{0} \checkmark$$

Suppose $\vec{u}, \vec{v}$ sol'ns to $A\vec{x} = \vec{0}$

& $\vec{p}$ is a soln to $Ax = \vec{b}$

Claim $s\vec{u} + t\vec{v} + \vec{p}$

is a soln to non-homog $A\vec{x} = \vec{b}$

Check:

$$A(s\vec{u} + t\vec{v} + \vec{p})$$

$$= A(s\vec{u} + t\vec{v}) + A\vec{p}$$

$$= \vec{0} + A\vec{p}$$

$$= \vec{0} + \vec{b} = \vec{b} \checkmark$$

no free variables

1.7: Linear Independence.

Defn: The set of vectors $\{\mathbf{a_1}, \mathbf{a_2}, ..., \mathbf{a_n}\}$ is **linearly independent** if and only if the equation $c_1\mathbf{a_1} + c_2\mathbf{a_2} + ... + c_n\mathbf{a_n} = \mathbf{0}$ has only the trivial solution.

$\underbrace{[\vec{a}_1 \ \vec{a}_2 \cdots \vec{a}_n]}_{A} \vec{x} = 0$

unique sol'n $\vec{x} = 0$ i.e.

$\Rightarrow$

Defn: If $\mathcal{S}$ is not linearly independent, then it is **linearly dependent**. $\Leftrightarrow$ $A\vec{x} = 0$ has ∞ # of sol'ns

free variables

Is $\left\{\begin{bmatrix} 9 \\ 7 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \end{bmatrix}\right\}$ is linearly independent?

$$\begin{bmatrix} 9 & 4 & 3 \\ 7 & 8 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ free variable

$\Rightarrow$ ∞ # of sol'ns

$\Rightarrow$ $\left\{\begin{bmatrix} 9 \\ 7 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \end{bmatrix}\right\}$ lin dep