22M:033 (MATH:2550) Engineering Math III: Matrix Algebra Fall 2014

Section 0001: 10:30A - 11:20A MW 101 BCSB

Section 0121: 12:30P - 1:20P MW 118 MLH

Instructor: Dr. Isabel Darcy *Office:* B1H MLH *Phone:* 335- 0778
Email: **idarcybiomath+33 AT gmail.com** or isabel-darcy AT uiowa.edu
Tentative Office Hours: MW 9:40 - 10:00am, 11:40 - 12:20, W 1:40pm - 2:40pm+, and by appt.
Note: + means I will be also available directly after the office hour, normally for as long as needed.

Click here for rest of syllabus including grading scheme

WebWork instructions (pdf), (doc format without comments).

TENTATIVE CLASS SCHEDULE-ALL DATES SUBJECT TO CHANGE (click on date/section for pdf file of corresponding class material):

Schedule		HW/Announcements
Week 1		
8/25	1.1, 1.2: Systems of Linear Equations, Row Reduction, and Echelon Forms , echelon practice	**NOT homework:** Practice Quiz 1: 1.1, 1.2 + Answers Echelon practice + answers
8/27	1.3: Vector Equations, ex, ex 1, ex 2	
Week 2		
9/1	Holiday: Labor Day	HW 1(**due date: Thursday 9/4**) Quiz 1(**Open Wed 8/27; Due: Sunday 9/7**)
9/3	1.3: Vector Equations	
Week 3		
9/8	1.4: The Matrix Equation Ax = b (No W drop date)	
9/10	1.5: Solution Sets of Linear Systems **Youtube lecture:** Solving homogeneous equations: Ax = 0 Putting answer in parametric vector form 18:27 minute video, pptx, pdf	
Week 4		

← Note

***** Thanksgiving break *****		
Week 14		
12/1	Review	
12/3	Exam 3: Ch 5, 6, and 7.1, answers	
Week 15		
12/8	7.2: Quadratic Forms	
12/10	7.2 ?? Review,FINAL EXAM INFO	
Final Exam Week		
TBA	Final Exam	Location: TBA

Note: In order to cover all the material needed for your engineering course, we have a very tight schedule. The schedule actually only allows for 2 review days and 2 exams. In order to squeeze time for an extra exam, I have created some youtube lectures. We will also have a series of youtube lectures during the 11th week of class when I will be in Mexico giving lectures on applications of topological data analysis. I will be available online to answer any questions, and a sub will be available during class times to provide additional help.

Resources for Students:

Math Tutorial Lab: 125 MLH
Engineering Tutoring Center: 3124 SC

+ me including e-mail

You will find links to my old exams on my older websites for this course: Spring 2014

Old exams from a 3 unit course I taught in 2002 covering more material:

MIDTERM #1 (Ch. 1, 2, 3.1 - 3.2, 9.9 from an old edition of Lay) , MIDTERM #1 answers

Problems we have covered so far this semester on old Midterm 1: 1, 5, 7, 12bcde

Review Exam 2, Exam 2 Answers (pdf), Exam 2 Answers (ps - better fonts?)

Problem set #1: Old midterm, Problem set #1 ANSWERS:

Exam 3, Exam 3 answers (pdf file), Exam 3 answers (ps file, better fonts?)

?? Final Exam Final Exam answers (pdf)

For some HW assignments, you may choose between 3 formats:

1.) Online HW. Log into Webwork for your HW problems. Note if you have difficulty with a problem click on e-mail instructor to send an e-mail describing the issue.

WebWork instructions (pdf), (doc format without comments).

2.) Paper version. Make sure you show all work.

3.) A combination of the above. Sometimes you might not be able to find that annoying addition error. you know how to do the problem, but can't get it correct on Webwork, turn in a paper version of that problem. Note also that on exams, you will be graded based on the work shown for the problem. If yoı would like your work graded on a HW problem, please turn in a paper version. Make sure to include, t problem number, the statement of the problem as well as the work you want graded. You can turn in either your individualized problem or the same problem number from the pdf/paper version.

On paper HW, you are required to check your answers when possible. If your answer is wrong, clearly indicate this on your HW. If you do not indicate this, you will receive 0 credit.

Note a homework or quiz grade below 93% strongly correlates with failing this class. When I taught this course in Spring 2014:

36.2% earned less then 93% on their homework. Of the students who earned less than 93% on their homework,
17.6% earned B's, 29.4% earned C's, 11.8% earned D's, 41.2% earned F's.

63.8% earned more than 93% on their homework. Of the students who earned more than 93% on their homework,
46.6% earned A's, 40% earned B's, and 13.3% earned C's.

Similar correlations held for quiz grades.

HW 1: Do one of the following:

1.) Online Webwork HW 1 (due date Thursday 9/4 at 11:59pm).
Note: Your user name = Your uiowa user name, but your password is your 8 digit HawkID (not your uiowa password). You should change your password after you log in.

2.) Paper Version of HW 1. Please check your answers

3.) A combination of the above (but you should do all 26 problems).

2M:033 (MATH:2550) Engineering Math III: Matrix Algebra Fall 2014

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Note: + means I will be also available directly after the office hour, normally for a long as needed.
DEO: Dan Anderson, 14 MLH, dan-anderson@uiowa.edu

Course WWW site:
http://www.math.uiowa.edu/~idarcy/COURSES/LinAlg/FALL14/2550.html
Check this for course schedule, assignments, possible course announcements, and changes, and electronic copies of course handouts.

Description of Course: This course is an abbreviated version of 22M:027. Here the emphasis is placed on matrices rather than on both linear transformations and matrices. Particular topics include operations on matrices, the use of matrix in solving systems of linear equations and evaluating determinants, eigenvalues and eigenvectors, the diagonalization of matrices and an introduction to subspaces of Euclidean space.

The engineering math sequence is for students in engineering. Students in the mathematical and physical sciences should take 22M:027 rather than 22M:033.

Prerequisites: Prerequisites: 22M:031 (MATH:1550).

Objectives and Goals of the Course: See course website for course schedule including a list of sections we will cover. This course focuses on methods and computations, but it lets you experience a bit of fairly abstract mathematics. The overall goals of the course are for you to understand the basic and major results in

All students should plan on being at the UI through the final examination period. Once the Registrar has announced the date, time, and location of each final exam, the complete schedule will be published on the Registrar's web site and will be shared with instructors and students. Do not make your end of the semester travel plans until the final exam schedule is made public. It is the student's responsibility to know the date, time, and place of the final exam.

Homework: We will not drop any homework grades, but you may replace 1 homework grade with a participation grade.

Quizzes: We will not drop any quiz grades, but you may replace up to 2 quiz grades with reviews of linear algebra resources as described at this link.

THERE IS NO CURVE IN THIS CLASS, but improvement may be taken in consideration.

NOTE
EXCELLENT HW grade $\xrightarrow[\text{usually}]{}$ above CLAS curve

If there is a mistake in grading, you must report this mistake within one week from when the exam, homework, etc. has been handed back to the class (whether or not you picked up your exam, homework, etc).

Attendance and absences: Your attendance at each scheduled class meeting is expected. You are responsible for material covered in class and announcements made during class; these may include changes in the syllabus.

Student Collaboration: You may collaborate with other students on the homework; however, each individual student is responsible for turning in your own homework in your own words. Copying is not collaboration and will be prosecuted under scholastic dishonesty. Any significant collaboration should be acknowledged.

The University policies on scholastic dishonesty will be strictly enforced.

Resources for Students:

Math Tutorial Lab: 125 MLH
Engineering Tutoring Center: 3124 SC

The College of Liberal Arts and Sciences Policies and Procedures

1.1, 1.2 Solving systems of linear equations.

Example: Solve

$$\begin{aligned} x + 2y + 3z &= 0 \\ 2x + 5y + 5z &= 4 \\ -x - 3y - 2z &= -4 \end{aligned}$$

$-2x \quad -4y \quad -6z \quad -0$

$$\begin{array}{cccc} x & y & z & \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2-2 & 5-4 & 5-6 & 4-0 \\ -1+1 & -3+2 & -2+3 & -4+0 \end{array}\right]$$

↓ eqn 2 - 2 eqn 1 → eqn 2,
↓ eqn 3 + eqn 1 → eqn 3

$$\begin{aligned} x + 2y + 3z &= 0 \\ 0x + y - z &= 4 \\ 0x - y + z &= -4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} \boxed{1} & 2 & 3 & 0 \\ 0 & 1 & -1 & 4 \\ 0 & -1+1 & 1-1 & -4+4 \end{array}\right]$$

↓ eqn 3 + eqn 2 → eqn 3

$$\begin{aligned} x + 2y + 3z &= 0 \\ 0x + y - z &= 4 \\ 0x + 0y + 0z &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} \boxed{1} & 2-2 & 3+2 & 0-8 \\ 0 & \boxed{1} & -1 & 4 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{array}\right]$$

↓ eqn 1 - 2eqn 2 → eqn 1

$$\begin{aligned} x + 0y + 5z &= -8 \\ 0x + y - z &= 4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & -8 \\ 0 & 1 & -1 & 4 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{array}\right]$$

depending on problem. Keep or delete row of all zero's

Thus $x = -8 - 5z$
$y = 4 + z$
$z = z$ (i.e., z is free, z can be any real number).

System of Linear Equations:

$$\begin{gathered}
a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \\
a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2 \\
\vdots \\
a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m
\end{gathered}$$

Coefficient Matrix:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & \vdots & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Augmented Matrix Form:

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ & & \vdots & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right]$$

Elementary row operations:

$$R_i \to cR_i \text{ where } c \neq 0$$

$$R_i \leftrightarrow R_j$$

$$R_i \to R_i + cR_j$$

$R_i \to \cancel{cR_i} + kR_j$

this affects the determinant

Two systems of equations are *equivalent* if they both have the same solution set.

$R_i \to cR_i \to cR_i + kR_j$

If two augmented matrices are row-equivalent, the corresponding linear systems of equations are equivalent.

Methods of solving a system of linear equations:

1.) Put matrix in Echelon Form

2.) Put matrix in Reduced Echelon form

Echelon form (non-unique):

The leftmost nonzero element in each row is called a *leading entry* or *pivot*.

i.) In each column with a leading entry, all entries below the leading entry are zero.

ii.) Each leading entry of a row is to the left of the leading entry of any row below it.

iii.) All rows of all zeros are below all non-zero rows.

(Note in echelon form, I do not require that the leading entry equal 1)

The position of a leading entry is called the *pivot position.*

A *pivot column* is a column containing a leading entry.

The variable corresponding to a pivot column is called a *basic variable.*

Variables that do not correspond to a pivot column are called *free variables.*

Determine if the augmented matrix is in echelon form. If it is, determine if the corresponding system of equations has no solution, exactly one solution, or an infinite number of solutions. If it has an infinite number of solutions, state the dimension of the hyperplane of the solutions.

EF

$$\begin{bmatrix} 5 & 6 & 3 & 7 & 2 \\ 0 & 7 & 5 & 2 & 8 \\ 0 & 0 & 4 & 1 & 4 \end{bmatrix}$$

$$\cancel{\begin{bmatrix} 0 & 6 & 3 & 7 & 2 \\ 0 & 7 & 5 & 2 & 8 \\ 0 & 0 & 4 & 1 & 4 \end{bmatrix}}$$

not ef

EF

$$\begin{bmatrix} 0 & 6 & 3 & 7 & 2 \\ 0 & 0 & 5 & 2 & 8 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

0's

ef

$$\begin{bmatrix} 0 & 6 & 3 & 7 & 2 \\ 0 & 0 & 5 & 2 & 8 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

EF

$$\begin{bmatrix} 0 & 6 & 3 & 7 & 2 \\ 0 & 0 & 5 & 2 & 8 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 6 & 3 & 7 & 2 \\ 0 & 0 & 5 & 2 & 8 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

EF

Rows of all zero's are at the bottom

EF
$$\begin{bmatrix} 3 & 6 & 3 & 7 & 2 \\ 0 & 9 & 5 & 2 & 8 \\ 0 & 0 & 7 & 1 & 4 \\ 0 & 0 & 0 & 5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 & 3 & 7 & 2 \\ 0 & 9 & 5 & 2 & 8 \\ 0 & 0 & 7 & 1 & 4 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
EF

EF
$$\begin{bmatrix} 3 & 6 & 3 & 7 & 2 \\ 0 & 9 & 5 & 2 & 8 \\ 0 & 0 & 7 & 1 & 4 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 & 3 & 7 & 2 \\ 0 & 9 & 5 & 2 & 8 \\ 0 & 0 & 7 & 1 & 4 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
EF

$$\cancel{\begin{bmatrix} 0 & 6 & 3 & 7 & 2 \\ 7 & 0 & 5 & 2 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}$$

$$\cancel{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 3 & 2 \\ 0 & 1 & 5 & 8 \end{bmatrix}}$$

Row-reduced echelon form (unique):

i.) The matrix is in echelon form.

ii.) The leading entries are all equal to 1.

iii.) In each column with a leading entry, all other entries are zero.

REQUIRED METHOD:

To put a matrix in echelon form work from left to right.

To put a matrix in row-reduced echelon form:

i.) First put in echelon form (work from left to right).

ii.) Put into reduced echelon form (work from right to left).

You may take short-cuts, but if your method is longer than the above, you will be penalized grade-wise.

Every matrix can be transformed by a finite sequence of elementary row operations into one that is in row-reduced echelon form.

Echelon form is not unique, but row-reduced echelon form is unique.

A system of LINEAR equations can have

i.) No solutions (inconsistent)

ii.) Exactly one solution

iii.) Infinite number of solutions.

Determine if the augmented matrix is in reduced echelon form. If it is, solve.

REF
$$\begin{bmatrix} 1 & 0 & 0 & 5 & 2 \\ 0 & 1 & 0 & 2 & 8 \\ 0 & 0 & 1 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 6 & 0 & 7 & 2 \\ 0 & 1 & 0 & 2 & 8 \\ 0 & 0 & 1 & 1 & 4 \end{bmatrix}$$
not EF
not REF

REF
$$\begin{bmatrix} 0 & 1 & 0 & 7 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$
not REF

REF
$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 0 & 1 & 5 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 5 & 0 & 8 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
leading entry $\neq 1$

REF

$$\begin{bmatrix} 1 & 0 & 5 & 7 & 0 & 0 & 2 \\ 0 & 1 & 3 & 4 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

REF

REF

$$\begin{bmatrix} 1 & 4 & 0 & 0 & 3 & 7 & 0 & 2 \\ 0 & 0 & 1 & 0 & 5 & 2 & 0 & 8 \\ 0 & 0 & 0 & 1 & 3 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

REF

$$\begin{bmatrix} 0 & 0 & 1 & 7 & 2 \\ 0 & 1 & 0 & 2 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

not EF

$$\begin{bmatrix} 3 & 6 & 3 & 7 & 2 \\ 0 & 9 & 5 & 2 & 8 \\ 0 & 0 & 7 & 1 & 4 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Solve:

$$3x + 6y + 9z = 0$$
$$4x + 5y + 6z = 3$$
$$7x + 8y + 9z = 0$$

$$\left[\begin{array}{cccc} 3/3 & 6/3 & 9/3 & 0/3 \\ 4 & 5 & 6 & 3 \\ 7 & 8 & 9 & 0 \end{array}\right]$$

$\downarrow$ $R1 \to R1/3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4^{-4} & 5^{-8} & 6^{-12} & 3^{-0} \\ 7^{-7} & 8^{-14} & 9^{-21} & 0^{-0} \end{array}\right]$$

$\downarrow$ $R_2 - 4R_1$, $R_3 - 7R_1$

$$\left[\begin{array}{ccc|c} \boxed{1} & 2 & 3 & 0 \\ 0 & -3 & -6 & 3 \\ 0^{-0} & -6^{+6} & -12^{+12} & 0^{+6} \end{array}\right]$$

$\downarrow$ $R_3 - 2R_2$

$R_2/-3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 6 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -3 & 3 \\ 4 & 5 & 6 & 3 \\ 7 & 8 & 9 & 0 \end{array}\right]$$

$R_2 - R_1 \to R_1$

affects determinant

optional method

No soln

$0x + 0y + 0z = 6$

$0 = 6$