Math 2418 Linear Algebra Exam #2 PART A Name: ____________________

October 31, 2001 SHOW ALL WORK Circle one: Wednesday/Thursday

[20] 1.) Find the QR-decomposition of $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \\ 0 & 3 & 10 \end{bmatrix}$.

Answer: $Q =$ ____________________ $R =$ ____________________

[2] 1b.) An orthonormal basis for the column space of A is ____________________.

2.) Let $W = span\{1+t,\ 1-3t\}$. Note that $\{1+t,\ 1-3t\}$ is an orthogonal set.
Using the inner product $< f,\ g >= \int_{-1}^{1} f(t)g(t)dt$, $< 1+t,\ 1+t >= \frac{8}{3}$ and $< 1-3t,\ 1-3t >= 8$.
Using this inner product, find the following:

[2] 2a.) $||1+t|| =$ ______________

[2] 2b.) $||1-3t|| =$ ______________

[3] 2c.) $< 7t^5,\ 1+t >=$ ______________

[3] 2d.) $< 7t^5,\ 1-3t >=$ ______________

[3] 2e.) If $\mathbf{v} = 7t^5$, $\quad proj_W \mathbf{v} =$ ______________

[3] 2f.) Is $7t^5$ in W? ______________

[3] 2g.) A vector in the orthogonal complement of W is ______________.

[4] 2h.) Find an orthogonal basis for $span\{1+t,\ 1-3t,\ 7t^5\}$ which includes $1+t$ and $1-3t$.

[2] 2i.) If $\mathbf{u} = t$, $\quad proj_W \mathbf{u} =$ ______________

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[18] 3a.) The following matrix has only one eigenvalue: $A = \begin{bmatrix} 2 & 3 & 3 & 3 \\ 0 & 2 & 1 & 8 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$.

Find the eigenvalue and a basis for the eigenspace corresponding to this eigenvalue

Answer 3a) Eigenvalue: $\lambda =$ ________

Basis for Eigenspace corresponding to λ:____________________________________

[3] 3b.) List 3 eigenvectors of A corresponding to λ:____________________________________

[3] 3c.) List two vectors in R^3 which are not eigenvectors of A: ________________________________

[12] 4.) Let $< (u_1, u_2),\ (v_1, v_2) > =\ 4u_1v_1 + 2u_1v_1.$
Show that this operation satisfies $< \mathbf{u} + \mathbf{v},\ \ \mathbf{w} > = < \mathbf{u},\ \ \mathbf{w} > + < \mathbf{v},\ \ \mathbf{w} >$

5.) Circle T for True or F for False.

[3] 5a.) If $\{\mathbf{v_1}, ..., \mathbf{v_n}\}$ is an orthogonal set of vectors, then $\{\mathbf{v_1}, ..., \mathbf{v_n}\}$ is linearly independent. T F

[3] 5b.) If λ is not an eigenvalue of A, then the linear system $(\lambda I - A)\mathbf{x} = \mathbf{0}$ has only the trivial solution T F

[3] 5c.) If the characteristic equation of A is $p(\lambda) = \lambda(\lambda - 5)(\lambda - 8)^2$, then A is invertible. T F

6.) Circle the correct answer

[3] 6a.) If W is a line in R^2, then $W^\perp$ is

i.) the empty set. ii.) a point. iii.) a line.

iv.) a 2-dimensional plane. v.) a 3-dimensional hyperplane.

[3] 6b.) If W is a line in R^3, then $W^\perp$ is

i.) the empty set. ii.) a point. iii.) a line.

iv.) a 2-dimensional plane. v.) a 3-dimensional hyperplane.

[3] 7a.) If $\mathbf{u}$ is in W, then $proj_W\mathbf{u} =$ _______________

[3] 7b.) If $\mathbf{u}$ is in $W^\perp$, then $proj_W\mathbf{u} =$ _______________