To define a homology, one only needs

1.) Objects = basis for R-module, R[objects]
2.) Grading
3.) Boundary map

Note (1) is an unneeded restriction. More generally, one only needs a chain complex, a sequence of abelian groups (or modules) connected by homomorphisms ∂ such that $\partial^2 = 0$:

$$... \xrightarrow{\partial} C_{n+1} \xrightarrow{\partial} C_n \xrightarrow{\partial} C_{n-1} \xrightarrow{\partial} ...$$

To create long exact sequences (of pair, of triple, meyer-vietoris), one only needs appropriate short exact sequences.

Thus homology is algebra, not topology.

You don't need any topology to do homology.

A homology theory requires that homology respect certain aspects of the topology of a space.

Categories:

1.) Topological spaces w/morphisms = continuous maps $f : X \to Y$.

2.) Chain complexes with morphism = chain maps:

$$\begin{tikzcd}
\cdots \arrow[r] & C_{n+1} \arrow[r, "\partial_{n+1}"] \arrow[d, "f_{n+1}"'] & C_n \arrow[r, "\partial_n"] \arrow[d, "f_n"'] & C_{n-1} \arrow[r] \arrow[d, "f_{n-1}"'] & \cdots \\
\cdots \arrow[r] & D_{n+1} \arrow[r, "\delta_{n+1}"] & D_n \arrow[r, "\delta_n"] & D_{n-1} \arrow[r] & \cdots
\end{tikzcd}$$

3.) $\bigoplus H_n(C)$ with morphism = homomorphism preserving grading.

Note we have covariant functors taking (1) to (2) to (3).

4.) $\bigoplus H^n(C)$ with morphism = homomorphism preserving grading.

Note we have a contravariant functor taking (1) to (4).

5.) $[X]$ such that $X_1 \sim X_2$ if there is a homotopy equivalence $h : X_1 \to X_2$ with morphisms $[f]$ where f is a continuous map and $f \sim g$ if f is homotopic to g.

Note H_n is a covariant functor taking (5) to (3), while H^n is a contravariant functor taking (5) to (4)