

Let  $p : E \rightarrow B$  be a covering map. Then

a.)  $p$  is an open map

b.)  $E$  locally (path) connected iff  $B$  locally path connected.

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Defn: A **separation** of  $X$  is a pair of nonempty open sets  $U, V$ , s.t.  $U \cap V = \emptyset$  &  $U \cup V = X$ .  $X$  is **connected** if there does not exist a separation of  $X$ .

Lemma:  $X$  is connected if and only if the only subsets of  $X$  which are both open and closed in  $X$  are the  $\emptyset$  and  $X$ .

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Defn: Given points  $x, y \in X$ , a **path** in  $X$  from  $x$  to  $y$  is a continuous map  $f : [a, b] \rightarrow X$  such that  $f(a) = x$  and  $f(b) = y$ .

A space is **path connected** if every pair of points of  $X$  can be joined by a path in  $X$ .

Lemma: Path connected implies connected.

Lemma 23.2: If  $C, D$  form a separation of  $X$  and if  $Y$  is a (path) connected subspace of  $X$ , then  $Y \subset C$  or  $Y \subset D$

Theorem 23.5: If  $f : X \rightarrow Y$  is continuous and  $X$  is (path) connected, then  $f(X)$  is (path) connected.

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Defn:  $X$  is **locally connected at  $x$**  if for every neighborhood  $U$  of  $x$ , there exists connected open set  $V$  such that  $x \in V \subset U$ .

$X$  is **locally connected** if  $x$  is locally connected at each of its points.

Defn:  $X$  is **locally path connected at  $x$**  if for every neighborhood  $U$  of  $x$ , there exists path connected open set  $V$  such that  $x \in V \subset U$ .

$X$  is **locally path connected** if  $x$  is locally path connected at each of its points.

Thm 25.3,4:  $X$  locally (path) connected iff  $\forall U$  open in  $X$ , each (path) component of  $U$  is open in  $X$

Thm 25.5: Each path component lies in a component of  $X$ .

If  $X$  is locally path connected, then the path components of  $X =$  components of  $X$ .