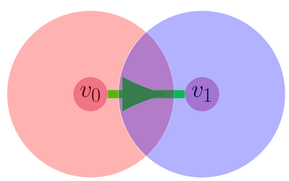
What does a generator of C_n embedded in a topological space X look like?

Let D^n be either
the closed $n-$dimensional disk $= \{x \in R^n \mid x \leq 1\}$ (for CW complex) or
the closed $n-$simplex $=$ convex hull of $n+1$ affinely independent points.

For a Cech complex: $\bigcap_{i=0}^{n} U_i$ is visualized via simplicial D^n.

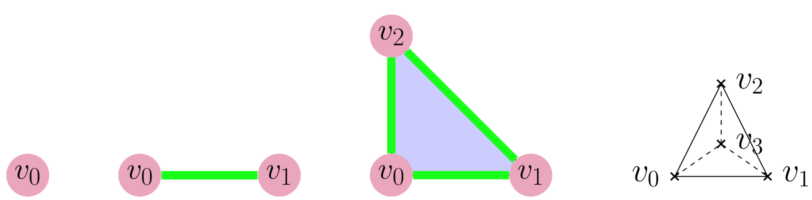


An abstract n-simplex:

$(v_0, ..., v_n)$ is visualized via simplicial D^n.

For a simplicial n-simplex:

$\Phi_\alpha : D^n_\alpha \to X$ is a homeomorphism.



For a Δ-simplex:

Assuming $\sigma_\beta : D^{n-1}_\beta \to X$ defined for all $n-1$-simplices,
$\sigma_\alpha : D^n_\alpha \to X$ is an extension of $\sigma_{\beta_i} : D^{n-1}_{\beta_i} \to X$
for all faces $D^{n-1}_{\beta_i}$ of D^n_α.
and $\sigma_\alpha : \overset{\circ}{D}{}^n_\alpha \to \sigma_\alpha(\overset{\circ}{D}{}^n_\alpha) \subset X$ is a homeomorphism.

Note that the restriction of the attaching maps
$\sigma_{\beta_i} : \overset{\circ}{D}{}^n_{\beta_i} \to \sigma_{\beta_i}(\overset{\circ}{D}{}^n_{\beta_i})$ are homeomorphisms.

$$\sqcup D^n \hookrightarrow X^{n-1} \cup \bigsqcup_\alpha D^n_\alpha \to X^{n-1} \cup \bigsqcup_\alpha D^n_\alpha / \sim \; = X^n \hookrightarrow X$$

For CW simplex:

The attaching maps $\phi_\alpha : \partial D^n_\alpha \to X^{n-1}$ are continuous.

The characteristic map $\Phi_\alpha : D^n_\alpha \to X$ extends
the attaching map $\phi_\alpha : \partial D^n_\alpha \to X^{n-1}$

and $\Phi_\alpha : \overset{\circ}{D}{}^n_\alpha \to \Phi_\alpha(\overset{\circ}{D}{}^n_\alpha) \subset X$ is a homeomorphism.

$$\sqcup D^n \hookrightarrow X^{n-1} \cup \bigsqcup_\alpha D^n_\alpha \to X^{n-1} \cup \bigsqcup_\alpha D^n_\alpha / \sim \; = X^n \hookrightarrow X$$

where $x \sim \phi_\alpha(x)$ for $x \in \partial D^n_\alpha$

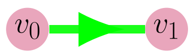
For a singular simplex: $\sigma_\alpha : D^n_\alpha \to X$ is a continuous map.

What does a generator of C_0 embedded in a topological space X look like?

- simplicial/Δ/CW/singular 1-simplex: A point. 

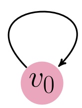
What does a generator (including boundary) of C_1 embedded in a topological space X look like?

- Simplicial 1-simplex:

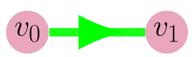


- Δ 1-simplex:

  Case 1: 1 vertex



  Case 2: 2 vertices

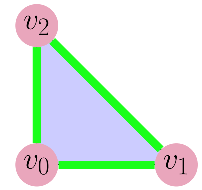


- CW 1-simplex: The same as a Δ complex.
- Singular 1-simplex: $f : [0, 1] \to X$, where f is continuous.

  I.e., a singular 1-simplex is a path in X. Note this includes the constant path.

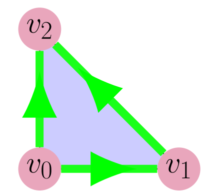
What does a generator (including boundary) of C_2 embedded in a topological space X look like?

- Simplicial 2-simplex:



- Δ 2-simplex:

  Each edge of the 2-simplex must be glued via an orientation preserving homeomorphism to a loop or edge of X^{n-1}.



  Case 1: 1 vertex

  Case 1a: 1 edge (Dunce hat)

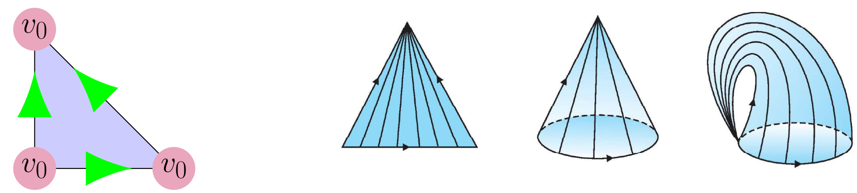
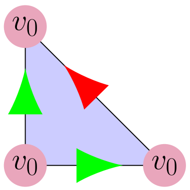


Figure 1: Identify the 2 circles. Dunce hat from math.stackexchange.com/questions/244885/dunce-hat-is-simply-connected by Ronnie Brown

See also Topological Dunce Hat by Jos Luis Rodrguez Blancas: https://youtu.be/34j4CppfRTA

Case 1bi: 2 edges



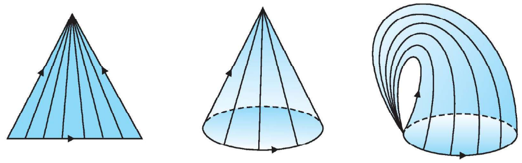
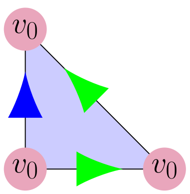
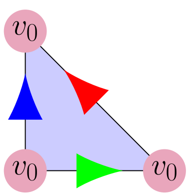


Figure 2: Don't identify the 2 circles math.stackexchange.com/questions/244885/dunce-hat-is-simply-connected by Ronnie Brown

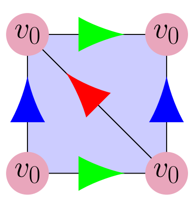
Case 1bii: 2 edges



Case 1c: 3 edges

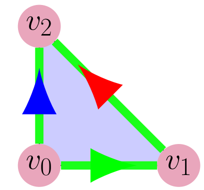


Example: torus



Case 2: 2 vertices

Case 3: 3 vertices.



Note each pair of distinct vertices defines a unique edge.

Example: Torus

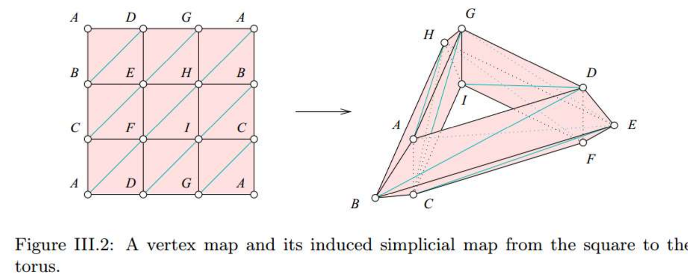


Figure 3: The simplicial triangulation of the torus is also a Δ-complex. From: Exterior, tex.stackexchange.com/questions/217645/typesetting-triangulations

- CW 2-simplex:

  Multiple possibilities.

  Only need attaching maps $\phi_\alpha : \partial D^2_\alpha \to X^1$ to be continuous where $D^2 = \{x \in R^2 \mid ||x|| \leq 1\}$

  For example,

- Singular 2-simplex: $f : \Delta^2 \to X$, where f is continuous.

  I.e., the image a singular 2-simplex is the image of a triangle in X. Note this image is a point if f is a constant map.