Lemma 27.5 (The Lebesgue number lemma)
If \( U \) is an open covering of the compact metric space \( X \), then \( \delta > 0 \) such that if \( A \subset X \) with \( \text{diam}(A) < \delta \), then \( \exists U \in U \) such that \( A \subset U \).

Thm 59.1: Suppose \( X = U \cup V \) where \( U, V \) are open and \( U \cap V \) is path connected. Let \( i_U : U \to X \) and \( i_V : V \to X \) be inclusion maps. Then \( \pi_1(X) \) is generated by the images of \( i_U^* \) and \( i_V^* \).

I.e., If \( g \in \pi_1(X) \), the \( g = g_1*g_2*...*g_n \) where for each \( i \), \( g_i \) is in either \( i_U^*(\pi_1(U)) \) or \( i_V^*(\pi_1(V)) \).

I.e., \( j : \pi_1(U) * \pi_1(V) \to \pi_1(X) \) induced by the two inclusion maps is surjective.

I.e, \( \pi_1(X) = \pi_1(U) * \pi_1(V) / \ker(j) \)

= \(< a_1, ..., a_i, b_1, ..., b_j | s_1, ..., s_l, t_1, ..., t_m > / \ker(j) >

Theorem 70.2. \( \ker(j) = \text{least normal subgroup} \)
generated by \( \{i_U(c_1)^{-1}i_V(c_1), ..., i_U(c_n)^{-1}i_V(c_n)\} \).

I.e., \( \pi_1(X) = < a_1, ..., a_i, b_1, ..., b_j | s_1, ..., s_l, t_1, ..., t_m, i_U(c_1)^{-1}i_V(c_1), ..., i_U(c_n)^{-1}i_V(c_n) > \)
The following maps are all induced by inclusion

\[ \pi_1(U) \xrightarrow{\pi_1(U \cap V)} \pi_1(X) \xrightarrow{\pi_1(V)} \]

Thm 70.1: \( U, V, U \cap V \) open and path-connected.