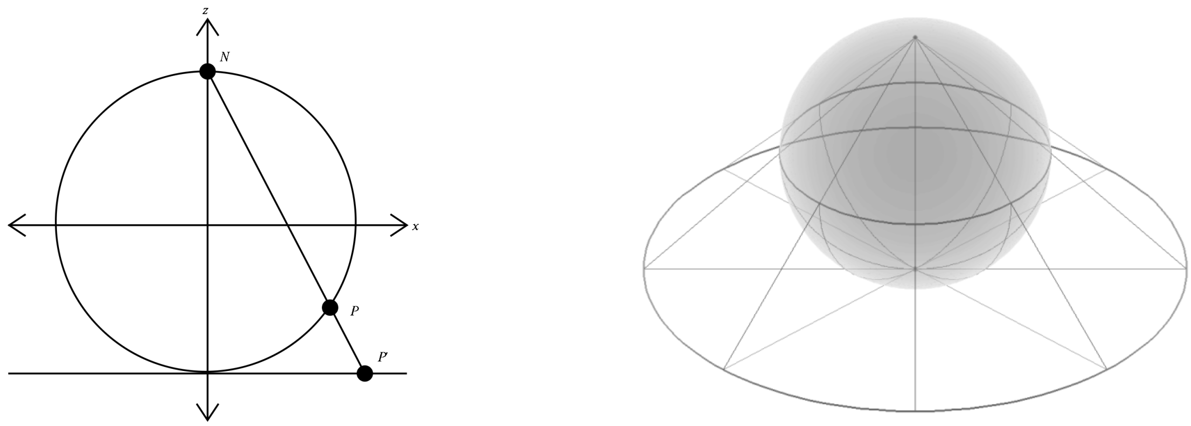
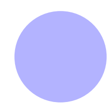
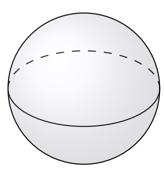
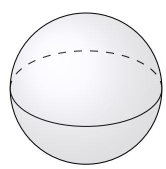


Figure 1: https://en.wikipedia.org/wiki/Stereographic_projection

Thus $S^1 = [\relbar\joinrel\relbar\joinrel\relbar] \bigcup_{\partial} [\relbar\joinrel\relbar\joinrel\relbar]$ and $S^2 =$ $\bigcup_{\partial}$ 

$S^3 =$  $\bigcup_{\partial}$ 

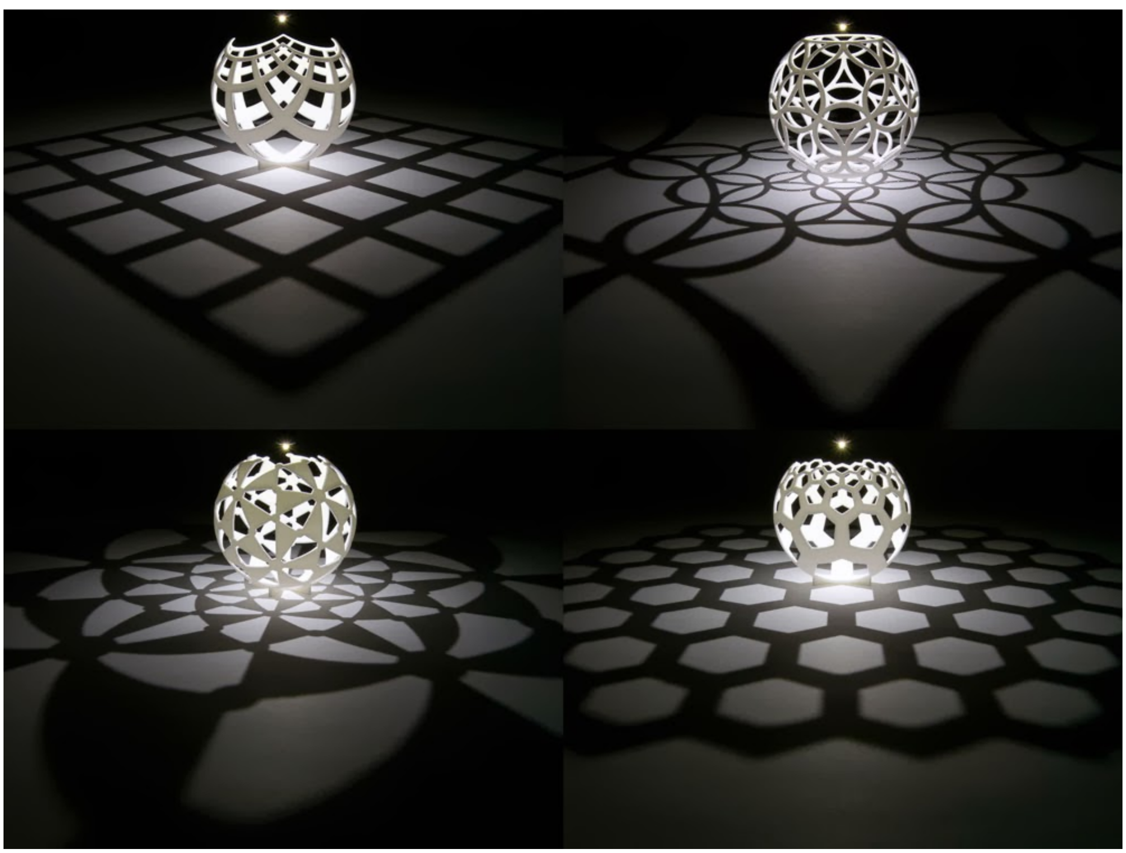


Figure 2: http://xahlee.info/math/i/stereographic_projection_by_Hen
12-12_95487.jpg

The following diagram does NOT commute:

$$
\begin{tikzcd}
\cdots \arrow[r] & C_{n+1} \arrow[r, "\partial_{n+1}"] \arrow[d, shift right, "f_{n+1}"'] \arrow[d, shift left, "g_{n+1}"] \arrow[dl, dashed] & C_n \arrow[r, "\partial_n"] \arrow[d, shift right, "f_n"'] \arrow[d, shift left, "g_n"] \arrow[dl, "K_n"'] & C_{n-1} \arrow[r] \arrow[d, shift right, "f_{n-1}"'] \arrow[d, shift left, "g_{n-1}"] \arrow[dl, "K_{n-1}"'] & \cdots \arrow[dl, dashed] \\
\cdots \arrow[r] & D_{n+1} \arrow[r, "\delta_{n+1}"] & D_n \arrow[r, "\delta_n"] & D_{n-1} \arrow[r] & \cdots
\end{tikzcd}
$$

The chain maps $f_n, g_n : C_n \to D_n$ are chain homotopic if $\exists K_n : C_n \to D_n$ such that

$$\delta_{n+1} K_n + K_{n-1} \partial_n = f_n - g_n$$

Claim: $f_* = g_* : H_\bullet^C \to H_\bullet^D$

Proof: If $\alpha \in Z_n^C$, then $\partial\alpha = 0$.

$\delta_{n+1} K_n(\alpha) + K_{n-1}\partial_n(\alpha) = f_n(\alpha) - g_n(\alpha)$

$f_n(\alpha) = \delta_{n+1} K_n(\alpha) + g_n(\alpha)$

$[f_n(\alpha)] = [\delta_{n+1} K_n(\alpha) + g_n(\alpha)] = [g_n(\alpha)]$

Thus $f_*(\alpha) = g_*(\alpha)$