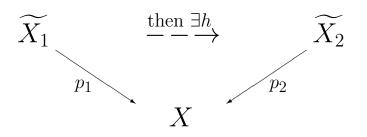
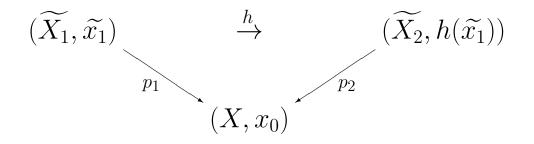
PROPOSITION 1.



Suppose  $p_1(\widetilde{x}_1) = p_2(\widetilde{x}_2) = x_0$  The covering maps  $p_1$ , and  $p_2$  are equivalent iff the subgroups  $(p_1)_*(\pi_1(\widetilde{X}_1, \widetilde{x}_1))$  and  $(p_2)_*(\pi_1(\widetilde{X}_2, \widetilde{x}_2))$  are conjugate in  $\pi_1(X, x_0)$ .

Proof:  $(\Rightarrow)$  Suppose h exists. Then by Prop 79.2,

$$(p_1)_*(\pi_1(\widetilde{X}_1, \widetilde{x}_1)) = (p_2)_*(\pi_1(\widetilde{X}_2, h(\widetilde{x}_1)))$$



By prop 79.3a,  $(p_2)_*(\pi_1(\widetilde{X}_2, h(\widetilde{x}_1)))$  and  $(p_2)_*(\pi_1(\widetilde{X}_2, \widetilde{x}_2))$  are conjugate in  $\pi_1(X, x_0)$ .

Thus  $(p_1)_*(\pi_1(\widetilde{X}_1, \widetilde{x}_1))$  and  $(p_2)_*(\pi_1(\widetilde{X}_2, \widetilde{x}_2))$  are conjugate in  $\pi_1(X, x_0)$ .

 $(\Leftarrow)$  Suppose  $H_1 = (p_1)_*(\pi_1(\widetilde{X}_1, \widetilde{x}_1))$  and  $H_2 = (p_2)_*(\pi_1(\widetilde{X}_2, \widetilde{x}_2))$  are conjugate in  $\pi_1(X, x_0)$ .

By 79.3b, there exists  $\widetilde{x'_2} \in \widetilde{X_2}$  such that  $H_1 = (p_2)_*(\pi_1(\widetilde{X_2}, \widetilde{x'_2}))$ Thus h exists by 79.2. DEFINITION 0.1. X is semilocally simply connected if  $\forall x \in X, \exists U \text{ open in } X \text{ such that } x \in U \text{ and the homomorphism induced by inclusion is trivial:}$ 

 $i_*: \pi_1(U, x) \to \pi_1(X, x), \qquad i([\alpha]) = [\alpha] = [e].$ 

**PROPOSITION** 2. If X has a universal cover, then X is semilocally simply-connected.

PROPOSITION 3. Suppose X is a path-connected, locally pathconnected, and semilocally simply-connected. Then for every subgroup  $H \subset \pi_1(X, x_0)$  there is a covering space  $p: \widetilde{X}_H \to X$ such that  $p_*(\pi_1(\widetilde{X}_H, \widetilde{x}_0)) = H$  for a suitably chosen basepoint  $\widetilde{x}_0 \in \widetilde{X}_H$ .

$$F: \left\{ \begin{array}{l} \widetilde{X} \\ p \\ \downarrow \\ X \end{array} \mid p \text{ is a covering map} \right\} \to \{H \mid H < G\}$$

 $F(p) = p_*(\pi_1(\widetilde{X}, \widetilde{x_0}))$ 

Note since  $p_* : \pi_1(\widetilde{X}, \widetilde{x_0}) \to \pi_1(X, x_0)$  is a homomorphism,  $p_*(\pi_1(\widetilde{X}, \widetilde{x_0}))$  is a subgroup of  $\pi_1(X, x_0)$ . Thus *F* is well-defined.

COROLLARY 1. X has a universal cover iff X is path-connected, locally-path connected, and semilocally simply-connected.