

NOTE: all maps are assumed to be continuous unless otherwise stated.

$r : X \rightarrow A$ is a retraction of X onto A
 if $r|_A = \text{identity map on } A$.

A is a retract of X if \exists a retraction of X onto A .

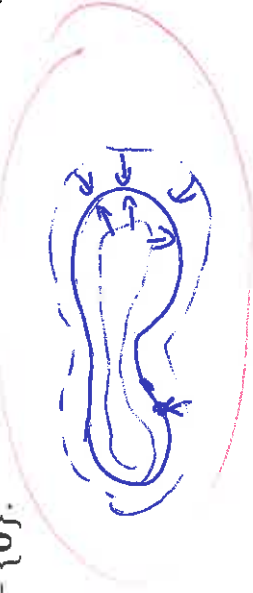
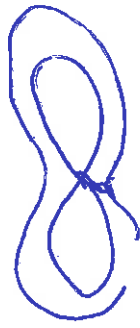
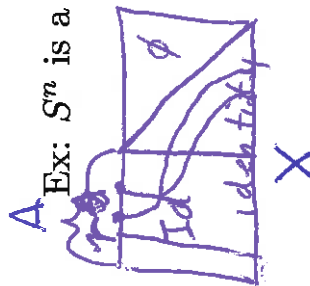
Ex: The constant map $c : X \rightarrow x_0$, where x_0 is a point of X is a retraction.

A is a deformation retract of X if the identity map $i : X \rightarrow X$ is homotopic to a map $R : X \rightarrow X$ where R is the extension (of the codomain) of a retraction $r : X \rightarrow A$ and where each point of A remains fixed during the homotopy.

In other words, \exists homotopy $H : X \times I \rightarrow X$ such that $H_0 = \text{identity map on } X$, $H_1(X) \subseteq A$, and $H_t|_A = \text{identity map on } A \forall t$.

Ex: X is a deformation retract of $X \times I$.

Ex: S^n is a deformation retract of $R^{n+1} - \{0\}$.



Lemma 55.1: If A is a retract of X , then $i_* : \pi_1(A) \rightarrow \pi_1(X)$ is injective where i_* is induced by the inclusion map.

Lemma 58.3: If A is a deformation retract of X , then $i_* : \pi_1(A) \rightarrow \pi_1(X)$ is an isomorphism where i_* is induced by the inclusion map.

Thm 70.1: Seifert-van Kampen Theorem.

Suppose U, V and $U \cap V$ are open and path-connected. Let $i : U \cap V \rightarrow U$ and $j : U \cap V \rightarrow V$ be inclusion maps. If

$$\pi_1(U) = \langle a_1, \dots, a_i \mid s_1, \dots, s_l \rangle,$$

$$\pi_1(V) = \langle b_1, \dots, b_j \mid t_1, \dots, t_m \rangle,$$

$$\pi_1(U \cap V) = \langle c_1, \dots, c_k \mid r_1, \dots, r_n \rangle,$$

then $\pi_1(U \cup V) = \langle a_1, \dots, a_i, b_1, \dots, b_j \mid s_1, \dots, s_l, t_1, \dots, t_m, i(c_1) = j(c_1), \dots, i(c_n) = j(c_n) \rangle$

