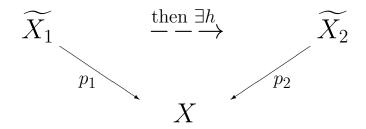
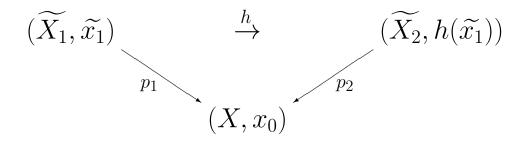
Proposition 1.



Suppose $p_1(\widetilde{x_1}) = p_2(\widetilde{x_2}) = x_0$ The covering maps p_1 , and p_2 are equivalent iff the subgroups $(p_1)_*(\pi_1(\widetilde{X_1},\widetilde{x_1}))$ and $(p_2)_*(\pi_1(\widetilde{X_2},\widetilde{x_2}))$ are conjugate in $\pi_1(X,x_0)$.

Proof: (\Rightarrow) Suppose h exists. Then by Prop 79.2,

$$(p_1)_*(\pi_1(\widetilde{X_1},\widetilde{X_1})) = (p_2)_*(\pi_1(\widetilde{X_2},h(\widetilde{X_1})))$$



By prop 79.3a, $(p_2)_*(\pi_1(X_2, h(\widetilde{x_1})))$ and $(p_2)_*(\pi_1(X_2, \widetilde{x_2}))$ are conjugate in $\pi_1(X, x_0)$.

Thus $(p_1)_*(\pi_1(\widetilde{X}_1, \widetilde{x}_1))$ and $(p_2)_*(\pi_1(\widetilde{X}_2, \widetilde{x}_2))$ are conjugate in $\pi_1(X, x_0)$.

 (\Leftarrow) Suppose $H_1 = (p_1)_*(\pi_1(\widetilde{X}_1, \widetilde{x}_1))$ and $H_2 = (p_2)_*(\pi_1(\widetilde{X}_2, \widetilde{x}_2))$ are conjugate in $\pi_1(X, x_0)$.

By 79.3b, there exists $\widetilde{x_2'} \in \widetilde{X_2}$ such that $H_1 = (p_2)_*(\pi_1(\widetilde{X_2}, \widetilde{x_2'}))$ Thus h exists by 79.2. DEFINITION 0.1. X is semilocally simply connected if $\forall x \in X, \exists U$ open in X such that $x \in U$ and the homomorphism induced by inclusion is trivial:

$$i_*: \pi_1(U, x) \to \pi_1(X, x), \quad i([\alpha]) = [\alpha] = [e].$$

PROPOSITION 2. If X has a universal cover, then X is semilocally simply-connected.

PROPOSITION 3. Suppose X is a path-connected, locally path-connected, and semilocally simply-connected. Then for every subgroup $H \subset \pi_1(X, x_0)$ there is a covering space $p \colon X_H \to X$ such that $p_*(\pi_1(X_H, \widetilde{x_0})) = H$ for a suitably chosen basepoint $\widetilde{x_0} \in X_H$.

COROLLARY 1. X has a universal cover iff X is path-connected, locally-path connected, and semilocally simply-connected.

$$F: \left\{ \begin{matrix} \widetilde{X} \\ p \\ X \end{matrix} \mid p \text{ is a covering map} \right\} \to \{H \mid H < G\}$$

$$F(p) = p_*(\pi_1(\widetilde{X}, \widetilde{x_0}))$$

Note since $p_*: \pi_1(\widetilde{X}, \widetilde{x_0}) \to \pi_1(X, x_0)$ is a homomorphism, $p_*(\pi_1(\widetilde{X}, \widetilde{x_0}))$ is a subgroup of $\pi_1(X, x_0)$. Thus F is well-defined.