Tutorial Session on Proof Techniques

GTCN 2014 teaching crew

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http://www.few.vu.nl/~rbakhshi/teaching/induction-handout.pdf
See also http://www.few.vu.nl/~rbakhshi/teaching/induction.pdf

Proof methods

Direct proof: Assume the condition, and prove the statement using known axioms, facts and theorems.

By construction: **If sufficient**, give example/counter-example.

By contradiction: Assume the opposite of the statement, and prove that it leads to obviously false claim.

By induction: In three stages: base case, hypothesis and induction step.

By (Mathematical) Induction

A task of the form:

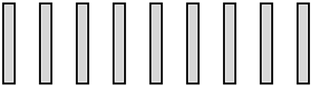
$$\textit{for } n \geq n_0, \textit{ show that } S(n).$$

A proof consists of these steps

- Base case: assuming $n = n_0$, show that $S(n_0)$ holds.
- Inductive step: Show that if $S(k)$ holds, then $S(k+1)$ holds.

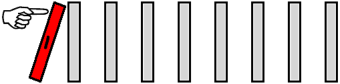
(see Note 2.14 of the coursebook)

Domino effect of Induction



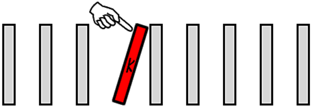
We have a statement $S(n)$ that we need to prove.

Base case: Proving $S(n_0)$ is like knocking down the first domino in the row:

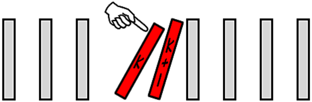


Show $S(n_0)$ is true.

Hypothesis: Assume for some $k \geq n_0$, $S(k)$ holds



Induction Step: Show that $S(k) \Rightarrow S(k+1)$



If we do all of the above, all the dominoes fall: $S(n)$ holds!

pics courtesy: Coolmath Algebra

(Mathematical) Induction

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$S(n)$ is a statement that depends on some integer $n > 0$.

Example (Integer n?)

- *size of* vertex set
- *size of* edge set
- *number of* cycles
- *number of* colors

Inductive Proof

What should inductive proof contain:

Induction basis: what induction is performed on, $S(n)$

Base case: the proof of $S(n_0) = \texttt{True}$ for the initial value of $n = n_0$

Induction step: assuming $S(k)$ holds, proof of $S(k+1) = \texttt{True}$. Also, how the induction hypothesis $S(k)$ is applied.

More on Induction Step

Assuming $S(k)$ holds, prove $S(k+1)$.

Example

Typical induction strategies include:

- Let e be an arbitrary edge in G, and let $G' = (V, E \backslash \{e\})$.
- Let v be an arbitrary vertex in G, and let G' be the subgraph of G obtained by deleting v and all its incident edges.

Induction on Graphs

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Exercise

Use **induction** to prove that if a *simple connected* graph G has at least 3 vertices, and each vertex is of degree 2, then it is a *cycle*.

Proof

$S(n)$ is $\{\forall$ connected graphs G, with $|V(G)| = n \geq 3$, where for all vertices $\forall v \in V : \delta(v) = 2$, G is a cycle$\}$. n is a *size* of the vertex set $|V(G)|$ of G.

Base case: $S(3)$: If $n = 3$, G is a triangle, and so, a cycle. $S(3) =$ True.

Ind. step: We prove that $S(k) \Rightarrow S(k+1)$.