(⇒) Suppose s is graphic.

Claim s^* is graphic.

s graphic implies $\exists$ simple graph $G = (V, E)$ with degree sequence s, where $V = \{v_1, ..., v_n\}$ where $\delta(v_i) = d_i$.

Let $G^* = G - v_1 = (V^*, E^*)$ where $V^* = \{v_2, ..., v_n\}$ ■ and $E^* = \{e \in E \mid e \text{ is not adjacent to } v_i\}$ v_1

Note G^* is a simple graph.

Case 1: Suppose v_1 is adjacent to $v_2, ..., v_{d_1+1}$

Removing v_1 and all edges adjacent to v_1 reduces the degree of $v_2, ..., v_{d_1+1}$ by one. Thus G^* has degree sequence s^*. Thus s^* is graphic if case 1 holds.

Case 2: Suppose $\exists j \in \{2, ..., v_{d_1+1}\}$ such that v_1 is NOT adjacent to v_j.

Then since $\delta(v_1) = d_1$, $\exists \ell > d_1 + 1$ such that v_1 is adjacent to v_ℓ.

See chalkboard for rest of proof.

More elegant proof of s is graphic implies s^* is graphic.

(⇒) Suppose s is graphic.

Claim s^* is graphic.

s graphic implies $\exists$ simple graph $G = (V, E)$ with degree sequence s, where $V = \{v_1, ..., v_n\}$ where $\delta(v_i) = d_i$.

Let $N(v_1) = \{w \in V \mid <v_1, w> \in E\}$

Let $A_G = N(v_1) \cap \{v_2, ..., v_{d_1+1}\}$

Note $0 \le |A_G| \le d_1$.

Among all simple graphs G with degree sequence s, choose one such that $|A_G|$ is as large as possible.

Let $G^* = G - v_1 = (V^*, E^*)$ where $V^* = \{v_2, ..., v_n\}$ ■ and $E^* = \{e \in E \mid e \text{ is not adjacent to } v_i\}$

Note G^* is a simple graph.

Case 1: Suppose $|A_G| = d_1$. Then v_1 is adjacent to $v_2, ..., v_{d_1+1}$

Then G^* has degree sequence s^*. Thus s^* is graphic. ■