$\forall$ = for all

$\exists$ = there exists

Defn

$f : A \to B$ is 1:1 iff $\boxed{f(x_1) = f(x_2)}$ implies $\boxed{x_1 = x_2}$.

$f(x_1) = f(x_2)$ implies $x_1 = x_2$.

Hypothesis: $f(x_1) = f(x_2)$. Conclusion $x_1 = x_2$.

$\boxed{\text{Hypothesis}}$ implies $\boxed{\text{conclusion.}}$
p implies q.
$p \Rightarrow q$.

Note a statement, $p \Rightarrow q$, is true if whenever the hypothesis p holds, then the conclusion q also holds.

To prove that a statement is true:
(1) Assume the hypothesis holds.
(2) Prove the conclusion holds.

Ex: To prove a function is 1:1:
(1) Assume $f(x_1) = f(x_2)$
(2) Do some algebra to prove $x_1 = x_2$.

$[p \Rightarrow q]$ is equivalent to $[\forall p,\ q$ holds$]$.

That is, for everything satisfying the hypothesis p, the conclusion q must hold.

Ex: $f(x) = 2x$
Assume $f(x_1) = f(x_2)$
$\Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$

A statement is false if the hypothesis holds, but the conclusion need not hold.

Hypothesis does not implies conclusion.
p does not imply q.
$p \not\Rightarrow q$.

That is there exists **a specific case** where the hypothesis holds, but the conclusion does not hold.

To prove that a statement is false:

Find an example where the hypothesis holds, but the conclusion does not hold.

Ex: To prove a function is not 1:1, find specific x_1, x_2 such that $f(x_1) = f(x_2)$, but $x_1 \neq x_2$.

Ex: $f : R \to R$, $f(x) = x^2$ is not 1:1 since $f(1) = 1^2 = 1 = (-1)^2 = f(-1)$, but $1 \neq -1$

$\sim [p \Rightarrow q]$ is equivalent to $\sim [\forall p,\ q$ holds$]$.

Thus if $p \Rightarrow q$ is false,
then it is not true that $[\forall p,\ q$ holds$]$.
That is, $\exists p$ such that q does not hold.