

HW 4: Due Wednesday Feb 6th

**Please check your answers.**

Please label all problems clearly (i.e., Q7, Q8, ...)

Please include your first and last name and the name of the assignment on your paper (e.g., "HW4").

Please write neatly and clearly, using full sentences. Practice pretending this is a real job for which **you want your HW to look professional.**

**Q 7:** Show that if every component of a graph is bipartite, then the graph is bipartite.

Note: answers in van Steen (and many if not most math books) are often incomplete (outlines as opposed to complete proofs). Please define all terms.

**Q 8:** Show that the complement of a bipartite graph need not to be a bipartite graph.

**Q 21:** Prove that if  $u$  is a vertex of odd degree in connected graph  $G$ , then there exists a path from  $u$  to another vertex  $v$  of  $G$  where  $v$  also has odd degree.

Note: van Steen gives 2 answers. First answer is best since it is shorter.

**Q 22:** Let  $d(u, v)$  denote the length of the shortest  $(u, v)$ -path in a connected graph  $G$ . Prove that  $d$  satisfies the triangle inequality: for any  $u, v, w \in V(G)$ :  $d(u, v) + d(v, w) \geq d(u, w)$ .

**Q 23:** Show that every simple graph with  $n$  vertices is isomorphic to a subgraph of the complete graph  $K_n$ .