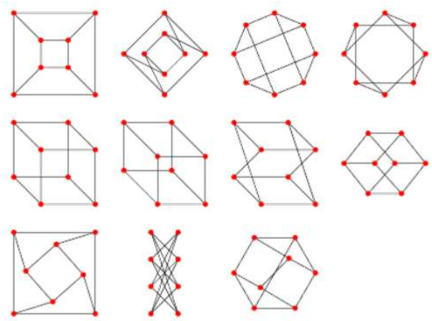
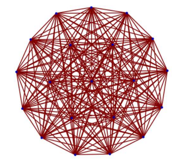
Drawing a graph



http://mathworld.wolfram.com/GraphEmbedding.html

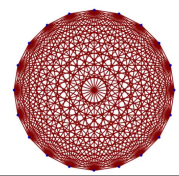
In[1]:= GraphPlot[Table[1, {20}, {20}]] https://reference.wolfram.com/language/ref/GraphPlot.html

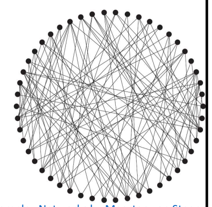
Out[1]=



In[2]:= GraphPlot[Table[1, {20}, {20}], Method → "CircularEmbedding"]

Out[2]=





Graph Theory and Complex Networks by Maarten van Steen

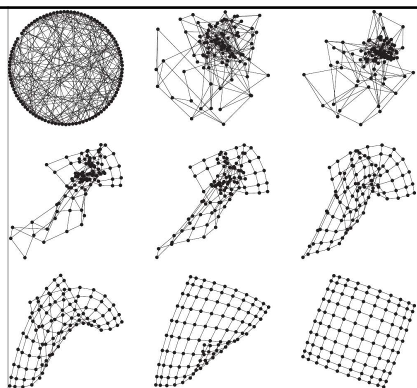
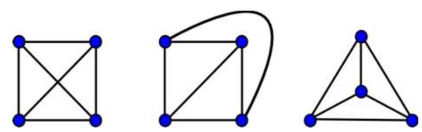


Figure 2.17: The evolution of applying a spring embedding to a graph.

Graph Theory and Complex Networks by Maarten van Steen

drp.math.umd.edu/Project-Slides/Characteristics of Planar Graphs.pptx

What is a planar embedding?

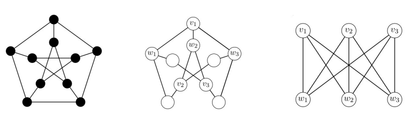


K_4

http://www.boost.org/doc/libs/1_49_0/libs/graph/doc/figs/planar_plane_straight_line.png

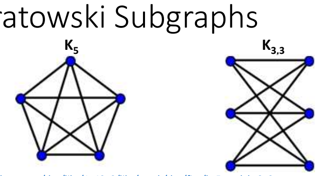
Kuratowski's Theorem (1930)

A graph is planar if and only if it does not contain a subdivision of K_5 or $K_{3,3}$.



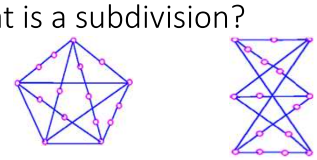
http://www.math.ucla.edu/~mwilliams/pdf/petersen.pdf

Kuratowski Subgraphs



http://www.boost.org/doc/libs/1_49_0/libs/graph/doc/figs/k_5_and_k_3_3.png

What is a subdivision?



Kuratowski Subgraphs

http://www.personal.kent.edu/~rmuhamma/GraphTheory/MyGraphTheory/Diagrams/g83.gif
http://www.personal.kent.edu/~rmuhamma/GraphTheory/MyGraphTheory/Diagrams/g82.gif

Euler characteristic (simple form):

χ = number of vertices – number of edges + number of faces

Or in short-hand,

$$\chi = |V| - |E| + |F|$$

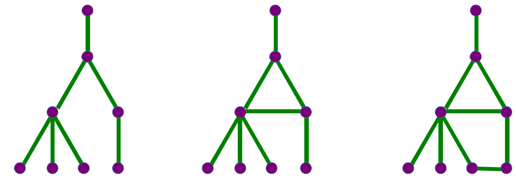
where V = set of vertices
E = set of edges
F = set of faces = set of regions

& the notation |X| = the number of elements in the set X.

For a planar connected graph $|V| - |E| + |F| = 2$

Defn: A *tree* is a connected graph that does **not** contain a cycle.

A *forest* is a graph whose components are trees.

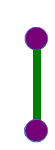


$\chi = 8 - 7 + 1 = 2$ $\chi = 8 - 8 + 2 = 2$ $\chi = 8 - 9 + 3 = 2$

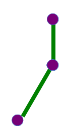
Lemma 2.1: Any tree with *n* vertices has *n-1* edges.

$\chi = |V| - |E| + |F|$

$\chi = 1 - 0 + 1 = 2$

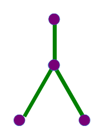


$\chi = 2 - 1 + 1 = 2$

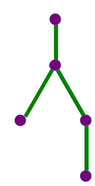


$\chi = 3 - 2 + 1 = 2$

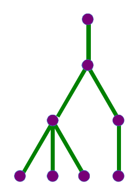
$\chi = |V| - |E| + |F|$



$\chi = 4 - 3 + 1 = 2$

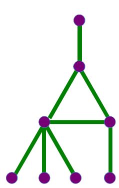


$\chi = 5 - 4 + 1 = 2$



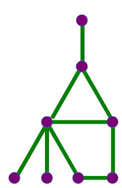
$\chi = 8 - 7 + 1 = 2$

$\chi = |V| - |E| + |F|$



$\chi = 8 - 8 + 2 = 2$

Not a tree.



$\chi = 8 - 9 + 3 = 2$

For the brave of heart, consider graphs drawn on other surfaces such as a torus or Klein bottle. For fun, see http://youtu.be/Q6DLWJX5tbs or www.geometrygames.org.

Euler's fomula: For a planar connected graph $|V| - |E| + |F| = 2$

where V = set of vertices, E = set of edges, F = set of faces = set of regions

Defn: A *tree* (or *acyclic graph*) is a connected graph that does **not** contain a cycle.

A *forest* is a graph whose components are trees.

Lemma 2.1: Any tree with *n* vertices has *n-1* edges.

Thm 2.9: For any connected planar graph with $|V| \geq 2$,

$$|E| \leq 3|V| - 6$$

Cor 2.4: K_5 is nonplanar.

Thm 2.10: $K_{3,3}$ is nonplanar.

Cor: A graph is planar if and only if it does not contain a subdivision of K_5 or $K_{3,3}$.