Math 3550 Quiz 1 Form A   Feb 7, 2020

1.) Circle T for true and F for false.

[4] 1a.) If $\vec{a}$ and $\vec{b}$ are vectors in $\mathbb{R}^3$, then $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$. T

[4] 1b.) The arc length s along the smooth curve with poisition vector $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$ from $\vec{r}(a)$ to $\vec{r}(b)$ is, by definition

$$s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}dt$$ T

2.) Determine whether or not the four points A(5, 2, -3), B(6, 4, 0), C(7, 5, 1), and D(14, 14, 18) are coplanar. If not find the volume of the parallelepiped spanned by $\overrightarrow{AB}$, $\overrightarrow{AC}$, and $\overrightarrow{AD}$.

$\overrightarrow{AB} = (6, 4, 0) - (5, 2, -3) = (1, 2, 3)$

$\overrightarrow{AC} = (7, 5, 1) - (5, 2, -3) = (2, 3, 4)$

$\overrightarrow{AD} = (14, 14, 18) - (5, 2, -3) = (9, 12, 21)$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 9 & 12 & 21 \end{vmatrix} = \begin{vmatrix} 1 & 2-2 & 3-3 \\ 2 & 3-4 & 4-6 \\ 9 & 12-18 & 21-27 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & -1 & -2 \\ 9 & -6 & -6 \end{vmatrix} = \begin{vmatrix} -1 & -2 \\ -6 & -6 \end{vmatrix} = (-1)(-6)-(-6)(-2) = 6-12 = -6$$

Note most row/column operations change the determinant. Make sure you know which does and which does not.

Thus the volume of the parallelepiped spanned by $\overrightarrow{AB}$, $\overrightarrow{AC}$, and $\overrightarrow{AD}$ is $|-6| = 6$.

Math 3550 Quiz 1 Form B   Feb 7, 2020

1.) Circle T for true and F for false.

[4] 1a.) The partial derivative value $f_x(a,b)$ is the slope of a line tangent to a curve on which y is constant and which passes through the point $(a, b, f(a,b))$ on the surface $z = f(a,b)$. T

[4] 1b.) The graph of the function $f(x,y) = 2 - 3x + 4y$ is a plane. T

[12] 2.) Find the unit tangent and normal vectors to the curve $y = x^3$ at the point (-1, -1).

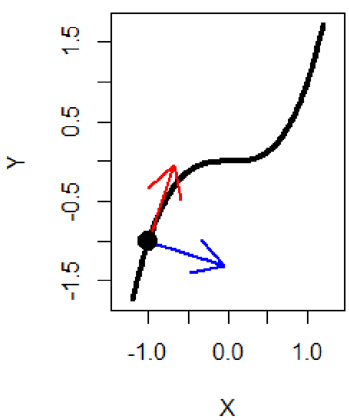
$y' = 3x^2$

$y'(-1) = 3(-1)^2 = 3$. Thus slope of tangent line at (-1, -1) is 3. Thus direction of tangent vector is (run, rise) $= (1,3) = (\frac{dx}{dx}, \frac{dy}{dx})$

Hence unit tangent vector $= \frac{(1,3)}{|(1,3)|} = \frac{(1,3)}{\sqrt{1^2+3^2}} = \frac{(1,3)}{\sqrt{10}} = (\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}})$

Normal vector is perpendicular to tangent vector.

Thus normal vector is either $(-\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}})$ or $(\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}})$ as these are both unit vectors whose dot product with the tangent vector is 0. By picture below, the unit normal vector is $(\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}})$



Thus unit tangent vector is $(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}})$

unit normal vector is $(\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}})$

Math 3550 Quiz 1 Form C Feb 7, 2020

1.) Circle T for true and F for false.

[4] 1a.) An equation for the plane through the three points (2, 4, -3), (3, 7, -1), (4, 3, 0) is $11x + y - 7z = 56$ F

$11(4) + 3 - 7(0) \neq 56$

[4] 1b.) If the cost function $C(x, y)$ of a box with base of length x and height y is given by

$$C(x, y) = 0.1(xy + \tfrac{100}{y} + \tfrac{100}{x})$$

then C is an independent variable and x and y are dependent variables. F

Note: x and y are independent variables and C is the dependent variable (as well as the name of the function).

[12] 2.) Find the arc length of the curve $x = sin(2t)$, $y = cos(2t)$, $z = 8t$ from $t = 0$ to $t = \pi$.

$$s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}dt = \int_0^\pi \sqrt{[2cos(2t)]^2 + [-2sin(2t)]^2 + [8]^2}dt$$

$$= \int_0^\pi \sqrt{4cos^2(2t) + 4sin^2(2t) + 64}dt = \int_0^\pi \sqrt{4 + 64}dt = \int_0^\pi \sqrt{68}dt = \sqrt{68}t|_0^\pi = \sqrt{68}\pi$$