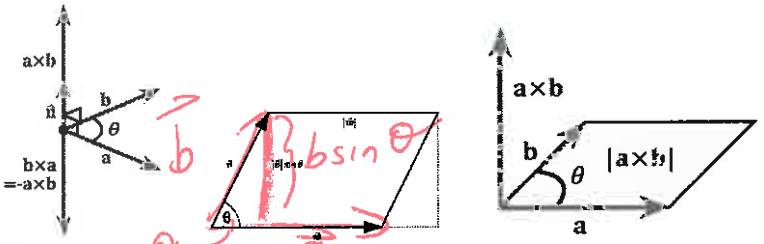
$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\ a_1&a_2&a_3\\ b_1&b_2&b_3\end{vmatrix}$$

$$\vec{a}\times\vec{b}=\begin{vmatrix}a_2&a_3\\ b_2&b_3\end{vmatrix}\vec{i}-\begin{vmatrix}a_1&a_3\\ b_1&b_3\end{vmatrix}\vec{j}+\begin{vmatrix}a_1&a_2\\ b_1&b_2\end{vmatrix}\vec{k}$$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\ a_1&a_2&a_3\\ b_1&b_2&b_3\end{vmatrix}\quad\begin{matrix}\vec{i}&\vec{j}\\ a_1&a_2\\ b_1&b_2\end{matrix}$$

$$\vec{a}\times\vec{b}=\begin{matrix}+\mathbf{i}a_2b_3\\ +a_3b_2\mathbf{k}\\ +b_3a_2\mathbf{k}\\ -b_2a_3\mathbf{k}\\ -\mathbf{i}b_2a_3\\ -a_2\mathbf{j}b_3\end{matrix}\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\ a_1&a_2&a_3\\ b_1&b_2&b_3\end{vmatrix}$$



Direction: Perpendicular to $\vec{a}$ and $\vec{b}$.

Length: $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|sin\theta$ = area of parallelogram with sides $\vec{a}$ & $\vec{b}$.

Color figures from: https://en.wikipedia.org/wiki/Cross_product,

BW figure from https://www.maa.org/sites/default/files/images/upload_library/4/vol6/Dray2/cross.gif

The volume of the parallelepiped is

$$\text{Volume} = |\mathbf{a}\times\mathbf{b}|\,|\mathbf{c}|\,|\cos\phi| = |(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}|.$$

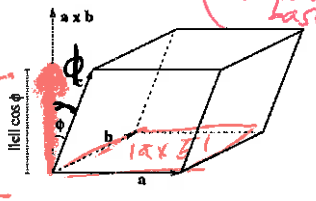


Figure: Nykamp DQ, The scalar triple product. From Math Insight. http://mathinsight.org/scalar_triple_product

Scalar triple product:

$$(\vec{a}\times\vec{b})\cdot\vec{c} = \left(\begin{vmatrix}a_2&a_3\\ b_2&b_3\end{vmatrix}\vec{i}-\begin{vmatrix}a_1&a_3\\ b_1&b_3\end{vmatrix}\vec{j}+\begin{vmatrix}a_1&a_2\\ b_1&b_2\end{vmatrix}\vec{k}\right)\cdot(c_1\vec{i}+c_2\vec{j}+c_3\vec{k})$$

$$= c_1\begin{vmatrix}a_2&a_3\\ b_2&b_3\end{vmatrix} - c_2\begin{vmatrix}a_1&a_3\\ b_1&b_3\end{vmatrix} + c_3\begin{vmatrix}a_1&a_2\\ b_1&b_2\end{vmatrix}$$

$$= \begin{vmatrix}a_1&a_2&a_3\\ b_1&b_2&b_3\\ c_1&c_2&c_3\end{vmatrix}$$