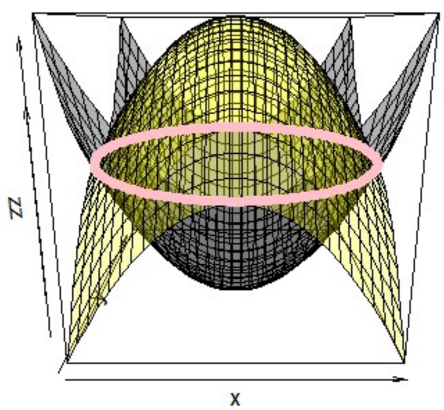
Find the volume between the surfaces: $z = x^2 + y^2 - 9$, $z = 16 - x^2 - y^2$

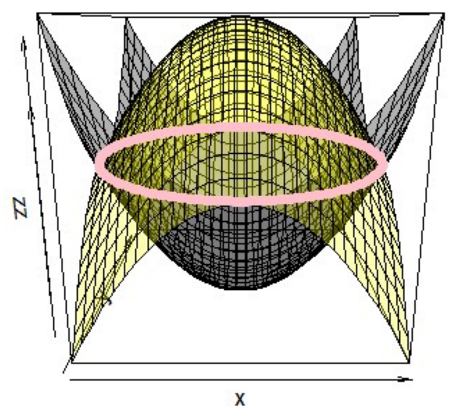
Need to find region over which to integrate:

Find intersection between surfaces: $x^2 + y^2 - 9 = 16 - x^2 - y^2$

Hence intersection is $2x^2 + 2y^2 = 25$

Thus (per figure) integrating over region $x^2 + y^2 \le \frac{25}{2}$





Height of columns: $16 - x^2 - y^2 - (x^2 + y^2 - 9) = 25 - 2x^2 - 2y^2$

Thus need to integrate $\int\int_R (25 - 2x^2 - 2y^2) dA$

Note: this integral is easier to compute using polar coordinates:

Height of columns: $25 - 2x^2 - 2y^2 = 25 - 2r^2$

Region: $x^2 + y^2 \le \frac{25}{2}$ is equivalent to $0 \le r \le \frac{5}{\sqrt{2}}$ and $0 \le \theta \le 2\pi$

Hence $\int\int_R (25 - 2x^2 - 2y^2)(dA) = \int_0^{2\pi} \int_0^{\frac{5}{\sqrt{2}}} (25 - 2r^2)(r dr d\theta)$

$= \int_0^{\frac{5}{\sqrt{2}}} \int_0^{2\pi} (25r - 2r^3) d\theta dr = \int_0^{\frac{5}{\sqrt{2}}} \int (25r - 2r^3)\theta|_0^{2\pi} dr = \int_0^{\frac{5}{\sqrt{2}}} (25r - 2r^3)(2\pi) dr$

$= (2\pi)(\frac{25}{2}r^2 - \frac{2}{4}r^4)|_0^{\frac{5}{\sqrt{2}}} = (\pi)[25(\frac{5}{\sqrt{2}})^2 - (\frac{5}{\sqrt{2}})^4)] = (\pi)[25(\frac{5^2}{2}) - (\frac{5^4}{4})]$

$= (\pi)[\frac{2(5^4) - 5^4}{4}] = (\pi)[\frac{5^4}{4}] = \frac{625\pi}{4}$

Find the volume between the surfaces: $z = x^2 + y^2 - 9$, $z = 16 - x^2 - y^2$

Use a triple integral:

If the density of this volume is $\delta(x, y, z) = x + y + z + 1$, find the mass of this volume.

The centroid is

$\overline{x} =$

$\overline{y} =$

$\overline{z} =$

Find the volume of the region bounded by $y^2 = x^2 + z^2$ and $y = 3$ using a triple integral.

Using Euclidean coordinates:

Integrate first with respect to z, then y, then x

Integrate first with respect to y, then z, then x

Note the volume of the region bounded by $y^2 = x^2 + z^2$ and $y = 3$ is the same as the volume of the region bounded by $z^2 = x^2 + y^2$ and $z = 3$

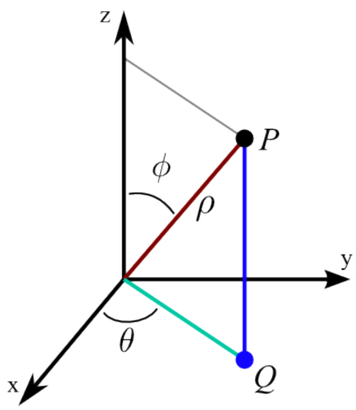
Use Cylindrical coordinates to find the volume of the region bounded by $z^2 = x^2 + y^2$ and $z = 3$.

Integrate first with respect to r, then θ, then z.

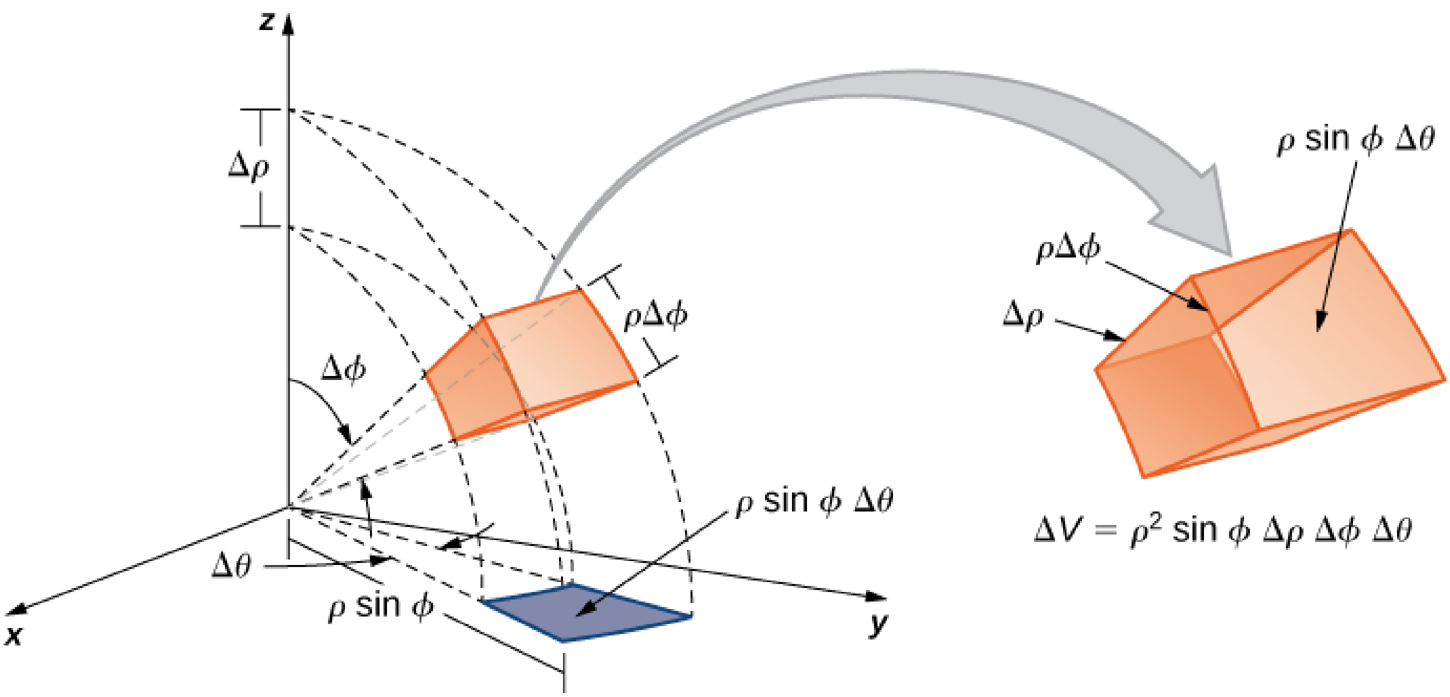
Integrate first with respect to z, then r, then θ

Spherical coordinates:

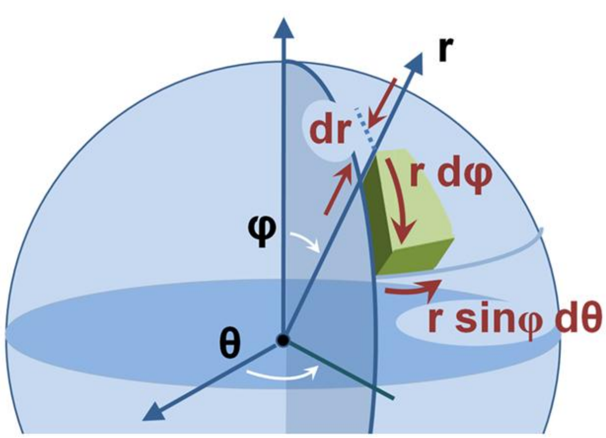
$$\begin{aligned} x &= \rho \sin\phi \cos\theta \\ y &= \rho \sin\phi \sin\theta \\ z &= \rho \cos\phi. \end{aligned} \tag{1}$$



https://mathinsight.org/spherical_coordinates



https://math.libretexts.org/Bookshelves/Calculus/Map%3A_Calculus_-_Early_Transcendentals_(Stewart)/15%3A_Multiple_Integrals/15.08%3A_Triple_Integrals_in_Spherical_Coordinates



https://en.wikipedia.org/wiki/File:Volume_element_spherical_coordinates.JPG

Use spherical coordinates to find the volume between spheres of radius 3 and radius 4.