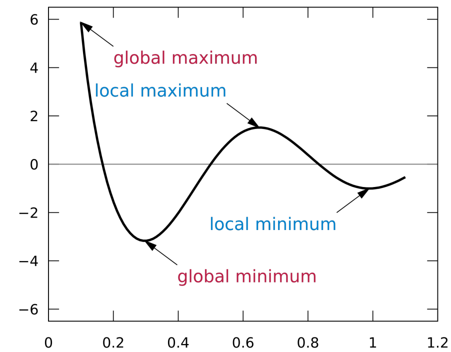
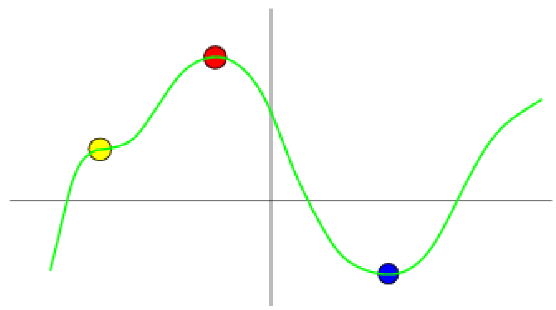
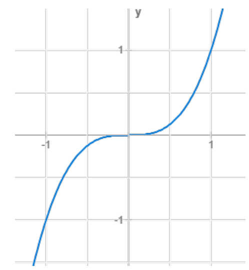
Calc 1 (one independent variables)

If $f(t_0)$ is a local maximum or local minimum, then $f'(t_0) = 0$ or DNE.







inflection point

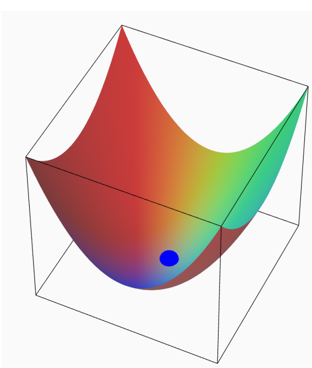
en.wikipedia.org/wiki/Maxima_and_minima mathinsight.org/local_extrema_introduction _two_variables bing.com

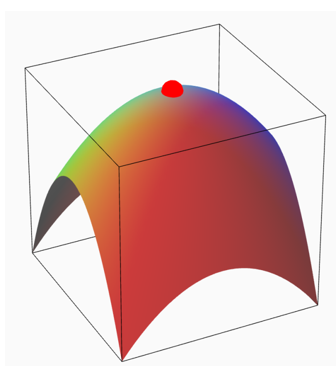
Extreme value theorem: If $f : [a, b] \to \mathbb{R}$ is continuous, then f must attain a maximum and a minimum, each at least once.

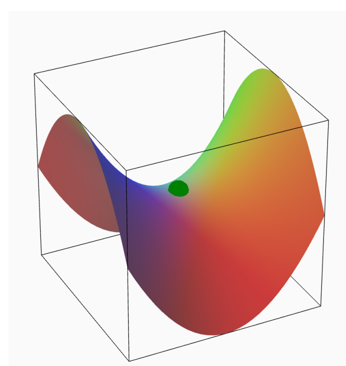
To find absolute max/min, check all t such that $f'(t) = 0$ or DNE as well as points on the boundary of $[a, b]$ (i.e., also check $f(a)$ and $f(b)$).

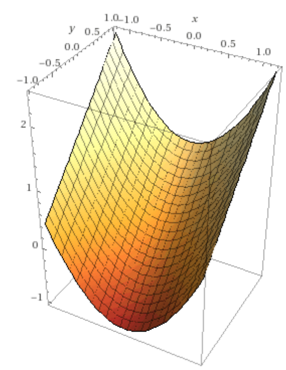
Section 12.5: Math 5 (multiple independent variables)

If $f(\mathbf{t_0})$ is a local maximum or local minimum, then for all x_i, $\frac{\partial f}{\partial x_i}(\mathbf{t_0}) = 0$ or DNE.



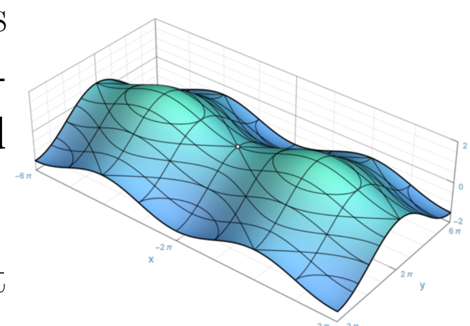






https://mathinsight.org/local_extrema_introduction_two_variables wolframalpha.com/ $z = x^2 + y$

Extreme value theorem: If $f : C \to \mathbb{R}$ is continuous where C is a closed and bounded region in $\mathbb{R}^n$, then f must attain a maximum and a minimum, each at least once.



To find absolute max/min, check all $\mathbf{t}$ such that for all x_i, $\frac{\partial f}{\partial x_i}(\mathbf{t_0}) = 0$ or DNE as well as points on the boundary of C.

https://en.wikipedia.org/wiki/Surface_(topology)

Example: Find the dimension of an open crate with volume 100 m^3 if material for bottom costs 10 cents/m^2 while the 4 sides cost 5 cents per m^2.

Solution: $lwh = 100$. Solve for one of the variables: $h = \frac{100}{lw}$

Cost $= 0.1lw + 0.5(2lh + 2wh = 0.1(lw + lh + wh)$

$C(l, w) = 0.1(lw + lh + wh) = 0.1(lw + l(\frac{100}{lw}) + w(\frac{100}{lw}))$

Thus $C(l, w) = 0.1(lw + \frac{100}{w} + \frac{100}{l})$

$\frac{\partial C}{\partial l} = 0.1(w - \frac{100}{l^2}) = 0$ or DNE. $\frac{\partial C}{\partial w} = 0.1(l - \frac{100}{w^2}) = 0$ or DNE.

Note $l \neq 0$ and $w \neq 0$ since volume $\neq 0$.

$0.1(l - \frac{100}{w^2}) = 0$ implies $l = \frac{100}{w^2}$

$0.1(w - \frac{100}{l^2}) = 0$ implies $0.1(w - \frac{100}{(\frac{100}{w^2})^2}) = 0.1(w - \frac{100w^4}{100^2}) = 0.1(w - \frac{w^4}{100}) = 0$

Thus $w(1 - \frac{w^3}{100}) = 0$. Hence $w = 0$ or $1 - \frac{w^3}{100} = 0$. Thus $w^3 = 100$.

Thus minimum occurs at $w = 100^{\frac{1}{3}} = 10^{\frac{2}{3}}$

$l = \frac{100}{w^2} = \frac{10^2}{10^{\frac{4}{3}}} = 10^{\frac{2}{3}}$ and $h = \frac{100}{lw} = \frac{10^2}{(10^{\frac{2}{3}})(10^{\frac{2}{3}})} = 10^{\frac{2}{3}}$

Thus dimension of box is $10^{\frac{2}{3}} \times 10^{\frac{2}{3}} \times 10^{\frac{2}{3}}$.

Cost is $0.1[lw + lh + wh] = 0.1[(10^{\frac{2}{3}})(10^{\frac{2}{3}}) + (10^{\frac{2}{3}})(10^{\frac{2}{3}}) + (10^{\frac{2}{3}})(10^{\frac{2}{3}})]$

Thus cost is \$ $0.3(10^{\frac{4}{3}}) = 0.3(10)(10^{\frac{1}{3}}) = 3(10^{\frac{1}{3}}) \sim 0.3(\frac{64}{3}) = \$6.40$

See 12.6 lecture to see that $\frac{64}{3}$ approximates $10^{\frac{4}{3}}$