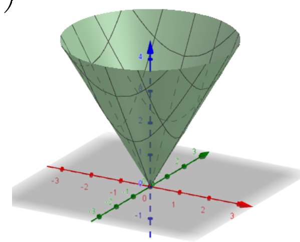
12.3: Limits and continuity

$z = f(\mathbf{x})$ is **continuous** at $\mathbf{x}_0$ if $\lim_{\mathbf{x}\to\mathbf{x}_0} f(\mathbf{x}) = f(\mathbf{x}_0)$

Ex: $\lim_{(x,y)\to(0,0)} \sqrt{3x^2+3y^2} =$

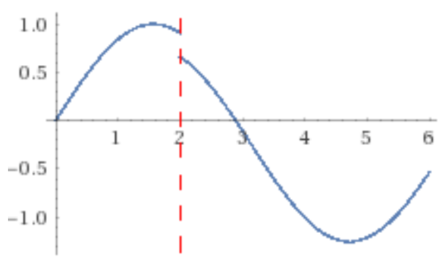


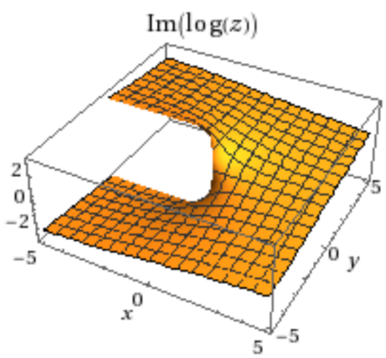
https://math.stackexchange.com/questions/2203478/triple-integrals-in-spherical-coordinates-z-sqrt3x23y2

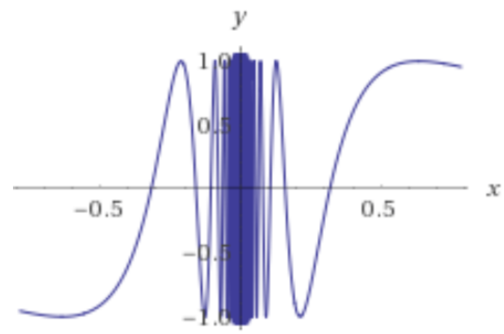
Defn: $\lim_{\mathbf{x}\to\mathbf{x}_0} f(\mathbf{x}) = L$ iff for all $\epsilon > 0$, there exists $\delta > 0$ such that if $|\mathbf{x} - \mathbf{x}_0| < \delta$, then $|f(\mathbf{x}) - L| < \epsilon$

***** In other words if $\mathbf{x}$ is close to $\mathbf{x}_0$, then $\mathbf{f}(\mathbf{x})$ is close to L *****

Discontinuous functions: $y = sin(\frac{1}{x})$







Understand 12.3: 51

Ex 9: Let $f(x,y) = \frac{xy}{x^2+y^2}$. $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} =$

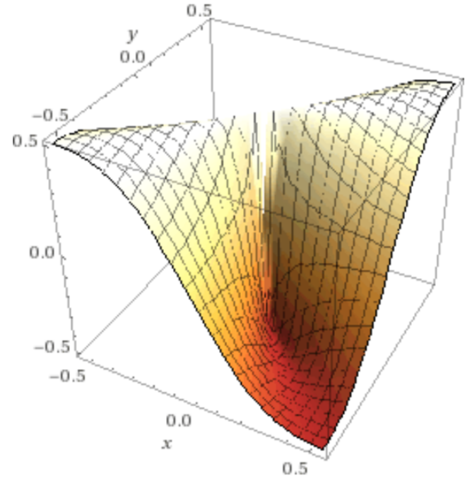
Suppose we travel to (0, 0) along a line with slope m. Thus $y = mx$:

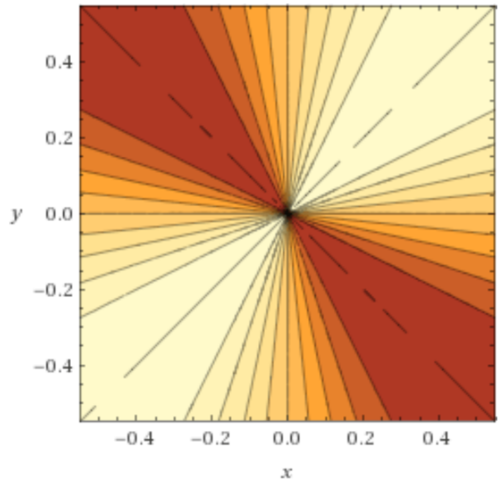
$$\lim_{(x,mx)\to(0,0)} \frac{xy}{x^2+y^2} = \lim_{(x,mx)\to(0,0)} \frac{x(mx)}{x^2+(mx)^2} = \lim_{x\to 0} \frac{mx^2}{(m^2+1)x^2}$$

$$= \lim_{x\to 0} \frac{m}{m^2+1} = \frac{m}{m^2+1}$$

Note $f(0,y) = 0$, $f(x,0) = 0$, $f(x,x) = \frac{1}{2}$, $f(x,-x) = -\frac{1}{2}$

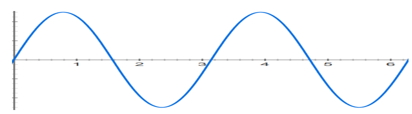
Ex 9: $z = \frac{xy}{x^2+y^2}$





Change to polar coordinates: Let $(x, y) = (rcos(\theta), rsin(\theta))$.

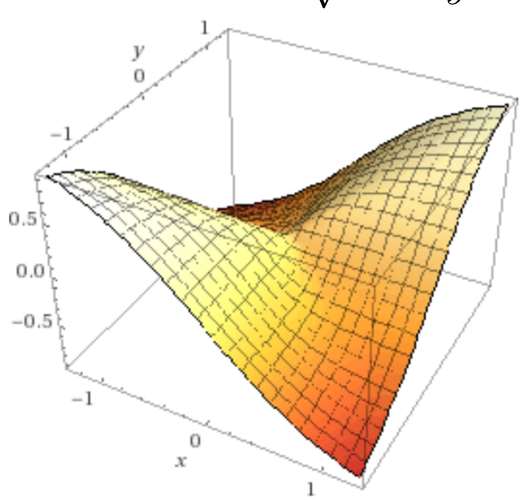
$\frac{xy}{x^2+y^2} = \frac{rcos(\theta)rsin(\theta)}{r^2} = cos(\theta)sin(\theta) = \frac{sin(2\theta)}{2}$

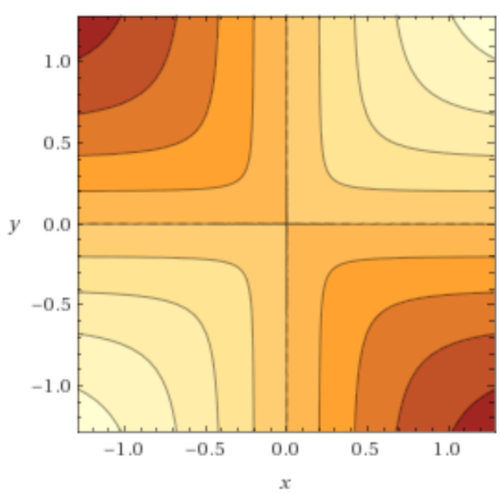


Thus $f(x, y) = \frac{xy}{x^2+y^2}$ in polar coordinates is

$$f(r, \theta) = \frac{sin(2\theta)}{2}, \quad r \geq 0,\ \theta \in [0, 2\pi]$$

Ex 8: $\lim_{(x,y)\to(0,0)} \dfrac{xy}{\sqrt{x^2+y^2}} =$





Change to polar coordinates: Let $(x, y) = (rcos(\theta), rsin(\theta))$.

$\frac{xy}{\sqrt{x^2+y^2}} = \frac{rcos(\theta)rsin(\theta)}{r} = rcos(\theta)sin(\theta) = \frac{rsin(2\theta)}{2}$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = \lim_{r\to 0} \frac{rsin(2\theta)}{2} =$$