2nd order linear homogeneous differential equation with NO damping
\[
\frac{d^2}{dx^2} y(x) + 49 y(x) = 0, \quad y(0) = 0, \quad D(y)(0) = 5
\]
General solution: \( y(t) = C_1 \sin(7.0t) + C_2 \cos(7.0t) \)
IVP solution: \( y(t) = 0.7142857143 \sin(7.0t) \)

Note initial conditions affect long term behaviour when there is NO damping. For example if \( y(0) = 1 \) and \( y'(0) = 5 \):
IVP solution: \( y(t) = 0.7142857143 \sin(7.0t) + \cos(7.0t) \)

2nd order linear NON-homogenous differential equation with NO damping
\[
y'' + 49y = 10\cos(7t), \quad y(0) = 0. \quad y'(0) = 5
\]
General solution: \( y(t) = \sin(7.0t) \) \( C_2 + \cos(7.0t) \) \( C_1 + 0.1020408163 \) \( \cos(7.0t) + 0.7142857143 \) \( \sin(7.0t) t \)
IVP solution: \( y(t) = 0.7142857143 \sin(7.0t) + 0.1020408163 \cos(7.0t) + 0.7142857143 \sin(7.0t) t \)

Do initial conditions have a long-term effect? Why?

\[
y'' + 49y = 10\cos(7t), \quad y(0) = 1. \quad y'(0) = 5
\]
IVP solution: \( y(t) = 0.7142857143 \sin(7.0t) + \cos(7.0t) + 0.7142857143 \sin(7.0t) t \)
2nd order linear NON-homogeneous differential equation with NO damping

\[ y'' + 49y = 10\cos(6t), \quad y(0) = 0, \quad y'(0) = 5 \]

General solution: \( y(t) = C_2 \sin(7.0\ t) + C_1 \cos(7.0\ t) + 0.7692307692\ \cos(6.0\ t) \)

IVP solution: \( y(t) = 0.7142857143\ \sin(7.0\ t) - 0.07692307692\ \cos(7.0\ t) + 0.07692307692\ \cos(6.0\ t) \)

Do initial conditions have a long-term effect? Why?

\[ y'' + 49y = 10\cos(6t), \quad y(0) = 1, \quad y'(0) = 5 \]

\[ y(t) = 0.7142857143\ \sin(7.0\ t) + 0.9230769231\ \cos(7.0\ t) + 0.07692307692\ \cos(6.0\ t) \]

2nd order homogeneous linear differential equation with DAMPING:

\[ y'' + y' + 49y = 0 \]

General solution: \( y(t) = C_1 e^{-0.5000000000\ t} \sin(6.982120020\ t) + C_2 e^{-0.5000000000\ t} \cos(6.982120020\ t) \)

\[ y'' + 14y' + 49y = 0 \]

General solution: \( y(t) = C_1 e^{-7.0\ t} + C_2 e^{-7.0\ t} t \)

\[ y'' + 15y' + 49y = 0 \]

General solution: \( y(t) = C_1 e^{-4.807417596\ t} + C_2 e^{-10.19258240\ t} \)

\[ y'' + 14y' + 49y = 0, \quad y(0) = 0, \quad y'(0) = 5 \]

IVP solution: \( y(t) = 5.0 e^{-7.0\ t} t \)

2nd order NON-homogeneous linear differential equation with DAMPING:

\[ y'' + 14y' + 49y = 100\cos(t), \quad y(0) = 0, \quad y'(0) = 5 \]

General solution: \( y(t) = C_2 e^{-7.0\ t} + e^{-7.0\ t} C_1 + 1.920000000\ \cos(t) + 0.5600000000\ \sin(t) \)

IVP solution: \( y(t) = -1.920000000\ e^{-7.0\ t} - 9.0 e^{-7.9\ t} + 1.920000000\ \cos(t) + 0.5600000000\ \sin(t) \)