$mr'' = \frac{-GMm}{r^2}$

Let $v = r'$, then $v' = r''$

Thus we obtain system of non-linear equations:

$$r' = v$$
$$v' = \frac{-GM}{r^2}$$

Note $v' = \frac{-GM}{r^2}$ involves 3 variables: v, t, r

Eliminate t: $\quad v' = \frac{dv}{dt} = \frac{dv}{dr}\frac{dr}{dt} = \frac{dv}{dr}v$

Thus $mv' = \frac{-GMm}{r^2}$ becomes $m\frac{dv}{dr}v = \frac{-GMm}{r^2}$

Separate variables: $\int mdvv = \int \frac{-GMm}{r^2}dr$

$$\frac{1}{2}mv^2 = \frac{GMm}{r} + E \text{ where } E \text{ is a constant.}$$

Thus we have derived the physics formula, conservation of energy:

$$\frac{1}{2}mv^2 + \frac{-GMm}{r} = E$$

I.e., Kinetic Energy + Potential Energy = constant

$x' = -3x + 12y$
$y' = -2x + 7y$

Suppose the following represent direction fields of linear systems of 1rst order differential equations in the phase plane. What can you say about solutions to these systems of equations.