

Integration by parts:

Derivative of a product: $(uv)' = uv' + vu'$

$$uv' = (uv)' - vu'$$

$$\int uv' = \int (uv)' - \int vu'$$

$$\int uv' = (uv) - \int vu'$$

Example: $\int e^{2x} \sin(3x)$

Let $u = \sin(3x)$, $dv = e^{2x}$

then $du = 3\cos(3x)$, $v = \frac{1}{2}e^{2x}$

then $d^2u = -9\sin(3x)$, $\int v = \frac{1}{4}e^{2x}$

$$\int e^{2x} \sin(3x) = \frac{1}{2} \sin(3x) e^{2x} - \int \frac{3}{2} e^{2x} \cos(3x)$$

$$= \frac{1}{2} \sin(3x) e^{2x} - \left[\frac{3}{4} \cos(3x) e^{2x} - \int \frac{-9}{4} \sin(3x) e^{2x} \right]$$

$$\int e^{2x} \sin(3x) = \frac{1}{2} \sin(3x) e^{2x} - \frac{3}{4} \cos(3x) e^{2x} - \frac{9}{4} \int \sin(3x) e^{2x}$$

$$\frac{13}{4} \int e^{2x} \sin(3x) = \frac{1}{2} \sin(3x) e^{2x} - \frac{3}{4} \cos(3x) e^{2x}$$

$$\int e^{2x} \sin(3x) = \frac{4}{13} \left[\frac{1}{2} \sin(3x) e^{2x} - \frac{3}{4} \cos(3x) e^{2x} \right]$$

Optional Exercise: Calculate $\int e^x \cos(2x)$