Ch 6 theorems for exam 2

Section 6.1

Thm 6.1.2:

Hint: Show $\int_0^\infty e^{st} f(t)dt$ exists by showing $lim_{A\to\infty} \int_0^A e^{st} f(t)dt$ exists (ie converges to a finite number) for $s > a$.

Note $\int_0^\infty e^{st} f(t)dt = \int_0^M e^{st} f(t)dt + \int_M^\infty e^{st} f(t)dt$

Thm: The Laplace transform is a linear operator.

Hint: $\mathcal{L}(af(t) + bg(t)) = ...$ OR [$\mathcal{L}(af(t)) = ...$ and $\mathcal{L}(f(t) + g(t)) = ...$]

Section 6.2

Thm 6.2.1:

Hint: use integration by parts and let $dv = f'(t)$

Cor 6.2.2': $\mathcal{L}(f''(t)) = s^2\mathcal{L}(f(t)) - sf(0) - f'(0)$

Hint. Use that f'' is the derivative of f'. Let $g = f'$ and note $\mathcal{L}(g'(t)) = s\mathcal{L}(g(t)) - g(0)$

Table 6.2.1

Section 6.3

Thm 6.3.1

Hint: $\int_0^\infty h(t)dt = \int_0^c h(t)dt + \int_c^\infty h(t)dt$ and use u-substitution (let $u = t - c$).

Thm 6.3.2

Hint: Let $F(s) = \mathcal{L}(f(t))$. Use definition of Laplace transform to evaluate $\mathcal{L}(e^{ct}f(t))$ and $F(s - c)$.

Better Hint: Let $F(s) = \mathcal{L}(f(t)) = ...$ To calculate $F(s - c)$ evaluate $F(s)$ at $s - c$ (i.e. replace s with $s - c$). Use definition of Laplace transform to evaluate $\mathcal{L}(e^{ct}f(t))$.

NOT on exam:

Thm: If f is a bijective linear function, then f^{-1} is also a linear function.

Cor: $\mathcal{L}^{-1}$ is linear.

Cor 6.2.2

Hint: Use proof by induction.