Math 34 Differential Equations Exam #1 October 9, 2003

SHOW ALL WORK

Solve the following differential equations

[15] 1a.) 
$$t^2y' + 2ty = t \sin(t)$$
.

[15] 1b.) y'' - 4y' + 4y = 0, y(0) = 2, y'(0) = 3.

Answer 1b.) \_\_\_\_\_

[15] 1c.) y'' - 4y' + 4y = 5t + 1.

[15] 1d.)  $2y'' - 3y^2 = 0$ , y(0) = 1, y'(0) = 1..

Answer 1d.) \_\_\_\_\_

[5] 2.) Use Euler's formula to write  $e^{2+3i}$  is the form of a+ib.

[5] 3.) Draw the direction field for y' = y.

- 4.) Circle T for true and F for false.
- [3] 4a.) Suppose  $\psi_1$  and  $\psi_2$  are solutions to the linear equation, ay'' + by' + cy = g(t), then  $\psi_1 + \psi_2$  must also be a solution to ay'' + by' + cy = g(t).

T F

[3] 4b.) Suppose  $\psi_1$  and  $\psi_2$  are solutions to the equation,  $ay'' + by' + cy^2 = 0$ , then  $\psi_1 + \psi_2$  must also be a solution to  $ay'' + by' + cy^2 = 0$ .

T F

[3] 4c.) Suppose  $\psi_1$  is a solution to the linear equation, ay'' + by' + cy = g(t), and  $\psi_2$  is a solution to the linear equation, ay'' + by' + cy = f(t), then  $5\psi_1 + 3\psi_2$  must also be a solution to ay'' + by' + cy = 5g(t) + 3f(t).

T F

[3] 4d.) If p, q, and g are continuous, then there exists a unique solution to  $y'' + p(t)y' + q(t)y = g(t), y(t_0) = y_0, y(t_1) = y_1.$ 

T F

[3] 4e.) If p, q, and g are continuous, then there exists a unique solution to y'' + p(t)y' + q(t)y = g(t),  $y(t_0) = y_0$ ,  $y'(t_0) = y'_0$ .

T F

[3] 4f.) Given an initial value, there always exists a unique solution to any first order differential equation.

T F

- 5.) Choose one of the two following problems. Clearly indicate which problem you have chosen.
- 5A.) Suppose the equation  $\frac{dp}{dt} = \gamma p$  describes the population of field mice. If the population of field mice doubles in 10 years, how long will it take the population to quadruple.
- 5B.) Find the escape velocity for a body projected upward with an initial velocity  $v_0$  from a point 3R above the surface of the earth, where R is the radius of the earth. Neglect air resistance. Recall that the equation of motion is  $m\frac{dv}{dt} = -\frac{mgR^2}{(R+x)^2}$  where x is the distance from the earth's surface.