

6.5: Impulse functions

Unit impulse function = Dirac delta function is a generalized function with the properties

$$\delta(t) = 0, \quad t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

$$\text{Let } d_c(t) = \begin{cases} \frac{1}{2c} & -c < t < c \\ 0 & t \leq -c \text{ or } t \geq c \end{cases}$$

Note $\lim_{c \rightarrow 0} d_c(t) = 0$ if $t \neq 0$

$$\text{and } \lim_{c \rightarrow 0} \int_{-\infty}^{\infty} d_c(t) dt = \lim_{c \rightarrow 0} 1 = 1 = \int_{-\infty}^{\infty} \delta(t) dt$$

$$\sin(t) = \frac{e^{it} - e^{-it}}{2i} \quad \cos(t) = \frac{e^{it} + e^{-it}}{2}$$

$$\sinh(t) = \frac{e^t - e^{-t}}{2} \quad \cosh(t) = \frac{e^t + e^{-t}}{2}$$

$$[\sinh(t)]' = \quad [\cosh(t)]' =$$

$$\sinh(0) = \frac{e^0 - e^0}{2} = 0 \quad \cosh(0) = \frac{e^0 + e^0}{2} = 1$$

Define the * product on R^2 by

$$(x_1, y_1) * (x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 - y_1 x_2)$$

Note * is

1.) commutative:

$$(x_1, y_1) * (x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 - y_1 x_2) = (x_2 x_1 - y_2 y_1, x_2 y_1 - y_2 x_1) = (x_2, y_2) * (x_1, y_1)$$

2.) associative: $(f * g) * h = f * (g * h)$

3.) distributive w.r.t +: $f * (g_1 + g_2) = f * g_1 + f * g_2$

4.) $(x_1, y_1) * (0, 0) = (0, 0)$

Note $(0, 1) * (0, 1) = (-1, 0)$

6.6: The Convolution Integral

Defn: The convolution of f and g is the function $f * g$ defined by

$$(f * g)(t) = \int_0^t f(t-s)g(s)ds = \int_0^t f(s)g(t-s)ds$$

Note * is

1.) commutative: $f * g = g * f$

2.) associative: $(f * g) * h = f * (g * h)$

3.) distributive w.r.t +: $f * (g_1 + g_2) = f * g_1 + f * g_2$

4.) $f * 0 = 0 * f = 0$