Assignment 16 (due 11/6) 6.2: 22, 23; 6.3: 5, 8, 10

Assignment 17 (due 11/11)
6.3: 9, 13, 15, 24, 25, 28, 29

Find the inverse LaPlace transform of $\frac{5s+21}{s^2+3s+4}$

Look at the denominator first to determine if you should factor and use partial fractions

$s^2 + 3s + 4$: $b^2 - 4ac = 3^2 - 4(1)(4) = 9 - 16 < 0$

Hence s^2+3s+4 does not factor over the reals. Hence to avoid complex numbers, we won't factor it.

$s^2 + 3s + 4$ is not an $s^2 - a^2$ or an $s^2 + a^2$, so it must be an $(s-a)^2 + b^2$.

Hence we will complete the square:

$s^2 + 3s + \underline{\quad} - \underline{\quad} + 4 = (s + \underline{\quad})^2 - \underline{\quad} + 4$

Hence $\frac{5s+21}{s^2+3s+4} = \frac{5s+21}{(s+\frac{3}{2})^2+\frac{7}{4}}$

Must now consider the numerator. We need it to look like $s - a = s + \frac{3}{2}$ or $b = \sqrt{\frac{7}{4}}$ in order to use
$\mathcal{L}^{-1}(\frac{s-a}{(s-a)^2+b^2}) = e^{at}cosbt$
and/or $\mathcal{L}^{-1}(\frac{b}{(s-a)^2+b^2}) = e^{at}sinbt$

$$5s + 21 = 5(s + \tfrac{3}{2}) - \tfrac{15}{2} + 21 = 5(s + \tfrac{3}{2}) - \tfrac{27}{2}$$

$$= 5(s + \tfrac{3}{2}) - [\tfrac{27}{2}\sqrt{\tfrac{4}{7}}]\sqrt{\tfrac{7}{4}} = 5(s + \tfrac{3}{2}) - [\tfrac{27}{\sqrt{7}}]\sqrt{\tfrac{7}{4}}$$

Hence $\frac{5s+21}{s^2+3s+4} = \frac{5(s+\frac{3}{2})-[\frac{27}{\sqrt{7}}]\sqrt{\frac{7}{4}}}{(s+\frac{3}{2})^2+\frac{7}{4}}$

$$= 5[\frac{s+\frac{3}{2}}{(s+\frac{3}{2})^2+\frac{7}{4}}] - \frac{27}{\sqrt{7}}[\frac{\sqrt{\frac{7}{4}}}{(s+\frac{3}{2})^2+\frac{7}{4}}]$$

Thus $\mathcal{L}^{-1}(\frac{5s+21}{s^2+3s+4}) = 5e^{-\frac{3}{2}t}cos\sqrt{\frac{7}{4}}t - \frac{27}{\sqrt{7}}e^{-\frac{3}{2}t}sin\sqrt{\frac{7}{4}}t$ ∎

6.3: Step functions.

$$u_c(t) = \begin{cases} 0 & t < c \\ 1 & t \geq c \end{cases}$$

1.) Graph $u_c(t)$:

2.) Given f, graph $u_c(t)f(t-c)$:

3.) Calculate $\mathcal{L}(u_c(t)f(t-c))$ in terms of $\mathcal{L}(f(t))$:

Example: Find the LaPlace transform of

1.)
$$g(t) = \begin{cases} 0 & t < 3 \\ e^{t-3} & t \geq 3 \end{cases}$$

2.)
$$f(t) = \begin{cases} 0 & t < 3 \\ 5 & 3 \leq t < 4 \\ t-5 & t \geq 4 \end{cases}$$

Example: Find the inverse Laplace transform of $\frac{e^{-8s}}{s^3}$

4.) Calculate $\mathcal{L}(e^{ct}f(t))$ in terms of $F(s) = \mathcal{L}(f(t))$

Example: Use formula 6 (p. 304) to find the inverse LaPlace transform of $\frac{s-c}{(s-c)^2+a^2}$.