Calulus pre-requisites you must know.

Derivative = slope of tangent line = rate.

Integral = area between curve and x-axis (where area can be negative).

The Fundamental Theorem of Calculus: Suppose f continuous on $[a, b]$.

1.) If $G(x) = \int_a^x f(t)dt$, then $G'(x) = f(x)$.

$$\text{I.e., } \frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x).$$

2.) $\int_a^b f(t)dt = F(b) - F(a)$ where F is any antiderivative of f, that is $F' = f$.

Suppose f is cont. on (a, b) and the point $t_0 \in (a, b)$, Solve IVP: $\frac{dy}{dt} = f(t)$, $y(t_0) = y_0$

$$dy = f(t)dt$$

$$\int dy = \int f(t)dt$$

$y = F(t) + C$ where F is any anti-derivative of F.

Initial Value Problem (IVP): $y(t_0) = y_0$

$$y_0 = F(t_0) + C \text{ implies } C = y_0 - F(t_0)$$

Hence unique solution (if domain connected) to IVP:

$$y = F(t) + y_0 - F(t_0)$$

CH 2: Solve $\frac{dy}{dt} = f(t, y)$

1.1: Direction Fields *********************

*******Existence/Uniqueness of solution**********

Thm 2.4.2: Suppose the functions $z = f(t, y)$ and $z = \frac{\partial f}{\partial y}(t, y)$ are cont. on $(a, b) \times (c, d)$ and the point $(t_0, y_0) \in (a, b) \times (c, d)$, then there exists an interval $(t_0 - h, t_0 + h) \subset (a, b)$ such that there exists a unique function $y = \phi(t)$ defined on $(t_0 - h, t_0 + h)$ that satisfies the following initial value problem:

$$y' = f(t, y), \quad y(t_0) = y_0.$$

Thm 2.4.1: If p and g are continuous on (a, b) and the point $t_0 \in (a, b)$, then there exists a unique function $y = \phi(t)$ defined on (a, b) that satisfies the following initial value problem:

$$y' + p(t)y = g(t), \quad y(t_0) = y_0.$$

But in general, $y' = f(t, y)$, solution may or may not exist and solution may or may not be unique.

Ex 1: $y' = y' + 1$

Ex 2: $(y')^2 = -1$

IVP ex 3: $\frac{dy}{dx} = y(1 + \frac{1}{x})$, $y(0) = 1$

$\int \frac{dy}{y} = \int (1 + \frac{1}{x})dx \quad$ implies $\quad ln|y| = x + ln|x| + C$

$|y| = e^{x+ln|x|+C} = e^x e^{ln|x|} e^C = C|x|e^x = Cxe^x$

$y = \pm Cxe^x$ implies $y = Cxe^x$

$y(0) = 1: \quad 1 = C(0)e^0 = 0$ implies

IVP $\frac{dy}{dx} = y(1 + \frac{1}{x})$, $y(0) = 1$ has no solution.

See direction field created using
www.math.rutgers.edu/∼ sontag/JODE/JOdeApplet.html

Ex Non-unique: $y' = y^{\frac{1}{3}}$

$y = 0$ is a solution to $y' = y^{\frac{1}{3}}$ since $y' = 0 = 0^{\frac{1}{3}} = y^{\frac{1}{3}}$

Suppose $y \neq 0$. Then $\frac{dy}{dx} = y^{\frac{1}{3}}$ implies $y^{-\frac{1}{3}}dy = dx$

$\int y^{-\frac{1}{3}}dy = \int dx$ implies $\frac{3}{2}y^{\frac{2}{3}} = x + C$

$y^{\frac{2}{3}} = \frac{2}{3}x + C$ implies $y = \pm\sqrt{(\frac{2}{3}x + C)^3}$

Suppose $y(3) = 0$. Then $0 = \sqrt{(2 + C)^3}$ implies $C = -2.$

Thus initial value problem, $y' = y^{\frac{1}{3}}$, $y(3) = 0$, has 3 sol'ns:

$$y = 0, \quad y = \sqrt{(\tfrac{2}{3}x - 2)^3}, \quad y = -\sqrt{(\tfrac{2}{3}x - 2)^3}$$

2.4 #27b. Solve Bernoulli's equation,

$$y' + p(t)y = g(t)y^n,$$

when $n \neq 0, 1$ by changing it

$$y^{-n}y' + p(t)y^{1-n} = g(t)$$

when $n \neq 0, 1$ by changing it to a linear equation by substituting $v = y^{1-n}$

Solve $ty' + 2t^{-2}y = 2t^{-2}y^5$

Section 2.5: Solve $\frac{dy}{dt} = f(y)$

If given either differential equation $y' = f(y)$ OR direction field:

Find equilibrium solutions and determine if stable, unstable, semi-stable.

Understand what the above means.