

Solve: $\vec{x}' = \begin{bmatrix} 4 & 1 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\begin{bmatrix} 4 & 1 \\ 5 & 0 \end{bmatrix}$ has e. vectors $c_1 \begin{bmatrix} -1 \\ 5 \end{bmatrix}$ w/ e. value = -1

and e. vectors $c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ w/ e. value = 5

Thus general solution is

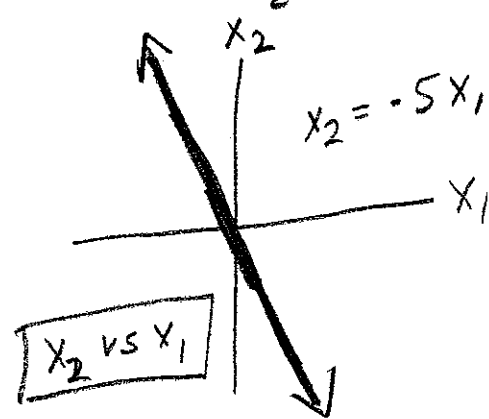
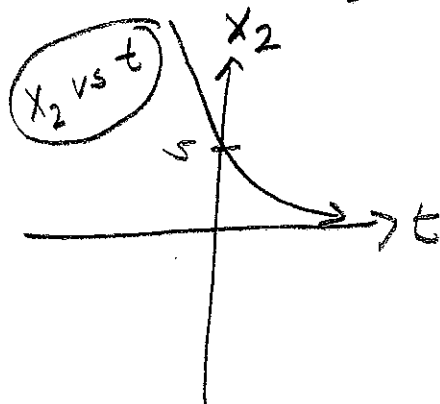
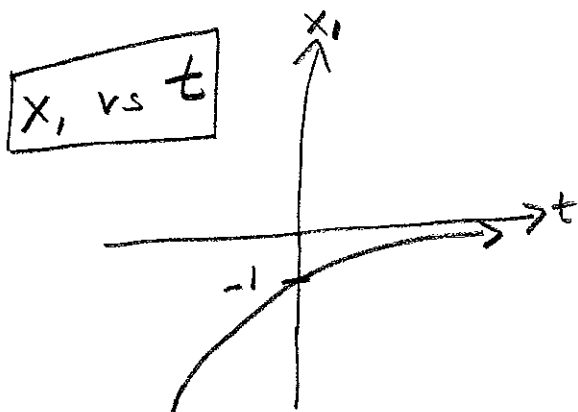
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = c_1 \begin{bmatrix} -1 \\ 5 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{5t}$$

I. V. P. : Suppose $\vec{x}(0) = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

$$\begin{bmatrix} -1 \\ 5 \end{bmatrix} = \vec{x}(0) = c_1 \begin{bmatrix} -1 \\ 5 \end{bmatrix} e^0 + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^0$$

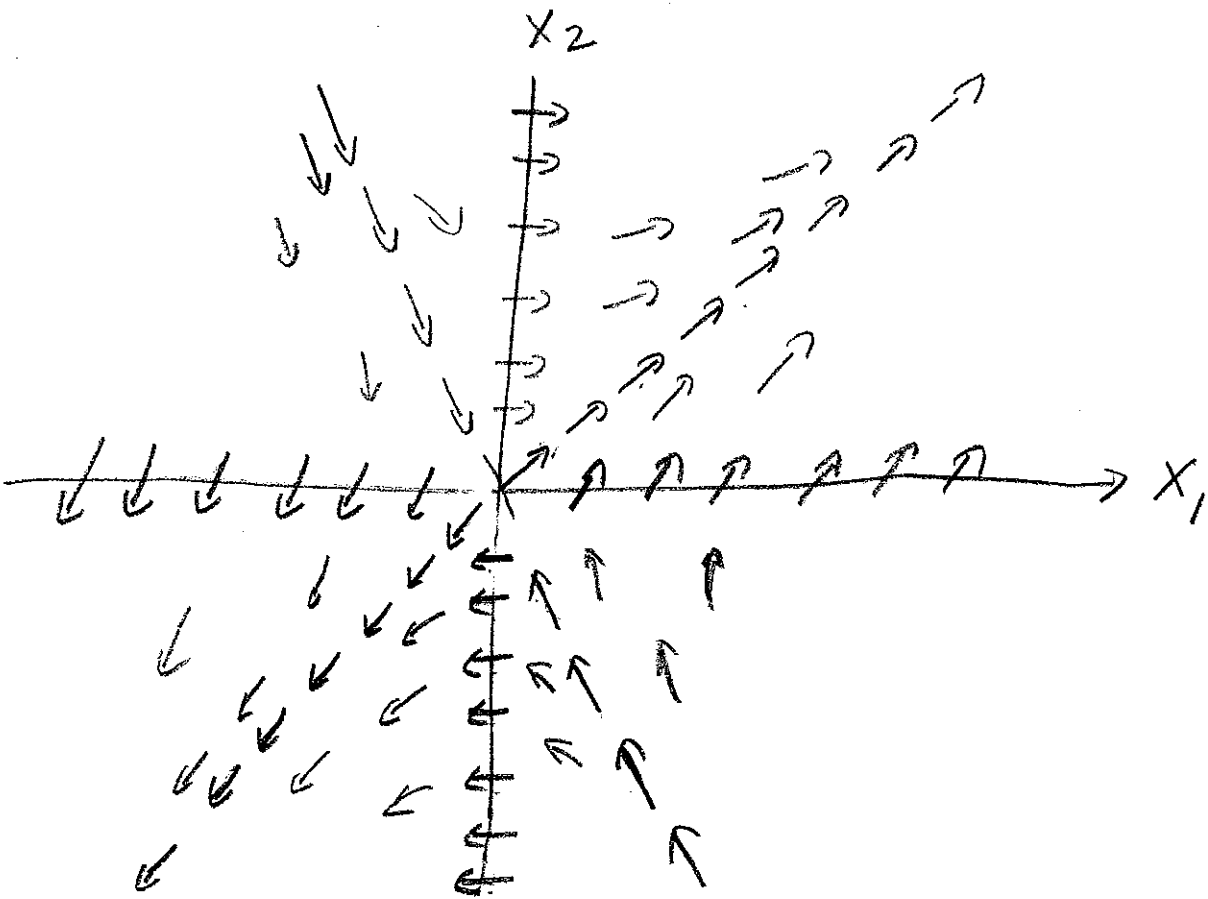
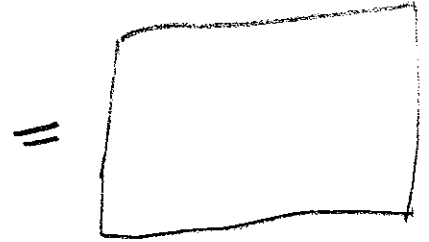
$$\left. \begin{aligned} -1 &= -c_1 + c_2 \\ 5 &= 5c_1 + c_2 \end{aligned} \right\} \Rightarrow c_1 = 1, c_2 = 0$$

If $\vec{x}(0) = \begin{bmatrix} -1 \\ 5 \end{bmatrix} \Rightarrow \vec{x} = \begin{bmatrix} -1 \\ 5 \end{bmatrix} e^{-t} \Rightarrow \begin{aligned} x_1 &= -e^{-t} \\ x_2 &= 5e^{-t} \end{aligned}$



$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 4x_1 + x_2 \\ 5x_1 \end{bmatrix}$$

$$\frac{dx_2}{dx_1} = \frac{dx_2}{dt} \cdot \frac{dt}{dx_1} = \frac{dx_2/dt}{dx_1/dt} = \frac{x_2'}{x_1'}$$



If $x_2 = -5x_1 \Rightarrow \frac{x_2'}{x_1'} = \frac{5x_1}{4x_1 - 5x_1} = \frac{5x_1}{-x_1} = -5$