Solve $N(U_f + \frac{0}{1}) = N(\frac{1}{0}) =$ unknot,
$N(U_f + \frac{t}{w}) = N(\frac{0}{1}) =$ unlink of two components.

Method 1:

1.) Since $N(\frac{0}{1})$ is a $(2, p)$ torus link, $N(\frac{1}{0}) =$ unknot, and E is rational, U_f is rational.

Suppose $U_f = \frac{a}{b}$. Then

$N(U_f + \frac{0}{1}) = N(\frac{a}{b} + \frac{0}{1}) = N(\frac{a}{b}) = N(\frac{1}{0})$.

Hence $a = 1$ since we can take a to be nonnegative $(\frac{a}{b} = \frac{-a}{-b})$.

Thus $U_f = \frac{1}{b}$.

$N(U_f + \frac{t}{w}) = N(\frac{1}{b} + \frac{t}{w}) = N(\frac{w+tb}{t}) = N(\frac{0}{1})$

Hence $w+tb = 0$. Thus $w = -tb$. Since $gcd(w, t) = 1$ and we can take t to be nonnegative, t $= 1$. Hence $w = -b$.

Thus the solutions to the above system of tangle equations is $U_f = \frac{1}{b}$ and $\frac{t}{w} = -\frac{1}{b}$ where b is an arbitrary integer.

Method 3: Use KnotPlot

Method 2:

Solve $N(U_f + \frac{0}{1}) = N(\frac{1}{0}) = N(\frac{a}{b}) = $ unknot,
$N(U_f + \frac{t}{w}) = N(\frac{0}{1}) = N(\frac{z}{v}) = 2$ component unlink.

$a = 1, b = 0, x = 0, y = -1, z = 0, v = 1,$
and v' is any integer such that $v'v^{\pm 1} = 1 \text{mod } z$

By corollary 2 if $w \ncong \pm 1$ mod t or if U_f is rational,
then $\frac{t}{w} = \frac{xz-av'}{bv'-yz-kt} = \frac{v'}{-kt} =$
and $U = \frac{a}{b+ka} = \frac{1}{k}$
Or $\frac{t}{w} = \frac{bz-av'}{xv'-yz-kt} = \frac{v'}{-kt} =$
and $U = \frac{a}{x+ka} = \frac{1}{k}$

By Thm 3, if $w \cong \pm 1$ mod t, $N(\frac{z}{v}) = N(\frac{tp(pb-qa) \pm a}{tq(pb-qa) \pm b})$

Hence $tp(pb - qa) \pm a = z$ or $-z$.
Thus $tp(pb - qa) = z \mp a$ or $-z \mp a$.

Hence $tp|(z \mp a)$.

IF $t \neq \pm 1$, then $U = (\frac{da-jb}{pb-qa} + \frac{j}{p}) \circ (h, 0)$ or $(\frac{j}{p} + \frac{da-jb}{pb-qa}) \circ (h, 0)$. Hence if $|z \mp a|$ is prime, U is rational.

If $t = \pm 1$, then U is rational by Hirasawa and Shimokaw

Solve $N(U'_f + \frac{f_1}{g_1}) = N(\frac{1}{0}) = N(\frac{a}{b}) =$ unknot,
$N(U'_f + \frac{f_2}{g_2}) = N(\frac{0}{1}) = N(\frac{z}{v}) = 2$ component unlink.

given that $U_f = -\frac{1}{k}$ and $\frac{t}{w} = \frac{1}{k}$ are the solutions to
$N(U_f + \frac{0}{1}) = N(\frac{1}{0}) = N(\frac{a}{b}) =$ unknot,
$N(U_f + \frac{t}{w}) = N(\frac{0}{1}) = N(\frac{z}{v}) = 2$ component unlink.

Suppose $\frac{f_1}{g_1} = (c_1, ..., c_n)$, $f_1 = E[c_1, ..., c_n]$, $g_1 = E[c_1, ..., c_{n-1}], e_1 = E[c_2, ..., c_n], i_1 = E[c_2, ..., c_{n-1}]$.

Then $\frac{f_2}{g_2} = \frac{te_1 + wf_1}{ti_1 + wg_1} = (b_1, ..., b_k + c_1, ..., c_n)$ where $\frac{t}{w} = (b_1, ..., b_k)$.

Since $\frac{t}{w} = \frac{1}{k} = (k, 0)$,
$\frac{f_2}{g_2} = \frac{e_1 + kf_1}{i_1 + kg_1} = (k, 0 + c_1, ..., c_n) = (k, c_1, ..., c_n)$.

$U'_f = U_f \circ (-c_1, ..., -c_n) = (-k, 0) \circ (-c_1, ..., -c_n) = (-k, 0 + -c_1, ..., -c_n) = (-k, -c_1, ..., -c_n)$

If $U_f = \frac{a}{b'}$, then $U'_f = U_f \circ (-c_1, ..., -c_n) = \frac{-f_1 b' + e_1 a}{g_1 b' - i_1 a}$.

Since $U_f = \frac{1}{-k}$, $U'_f = \frac{f_1 k + e_1}{g_1 k - i_1}$

Note: $(1) = (0,\ 1,\ 1)$, but
$(3) = (2) \circ (1) \neq (2) \circ (0, 1, 1) = (2, 1, 1)$