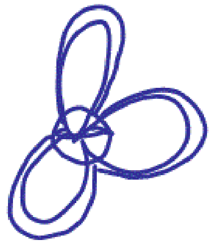
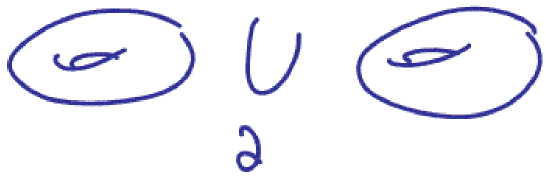
0.1 Heegaard Diagrams

Defn: A handlebody, U, of genus g is a regular neighborhood of a bouquet of g circles.

I.e, U = one 0-handle $\cup$ g 1-handles.

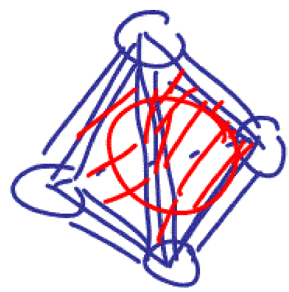


Defn: A Heegaard decomposition of a 3-manifold $Y = U_1 \cup_\Sigma U_2$ where U_i are genus g handlebodies and $\Sigma \quad \partial U_1 = \partial U_2$.



Thm: Every 3 manifold Y^3 has a Heegaard splitting.

Proof. : Every 3 manifold has a triangulation. □



Defn: Let U be a handlebody and let $\Sigma \quad \partial U$. A set of *attaching circles* $(\gamma_1, \ldots, \gamma_g)$ for U is a g-tuple of simple closed curves on Σ such that

(1) $\gamma_i \cap \gamma_j = \emptyset$ for $i \neq j$

(2) γ_is are linearly (i.e. homologically) independent in $H_1(\Sigma)$ [or equivalently $\Sigma - \cup\gamma_i$ is connected].

(3) γ_i bound disjoint embedded disks in U