

Feb 11: 4D10 Knots in Solid tori and Companionship.

Note Title

2/7/2010

Ex 4D3: Suppose L is a closed subset of a solid torus V , then TFAE:

- (a) L is geometrically essential (i.e., L intersects every meridional disk of V).
- (b) There does not exist a 3-ball B s.t. $L \subset B \subset V$.
- (c) The inclusion homomorphism $\pi_1(\partial V) \rightarrow \pi_1(V - L)$ is 1:1.



Let L be geometrically essential link in $V = S^1 \times D^2$.

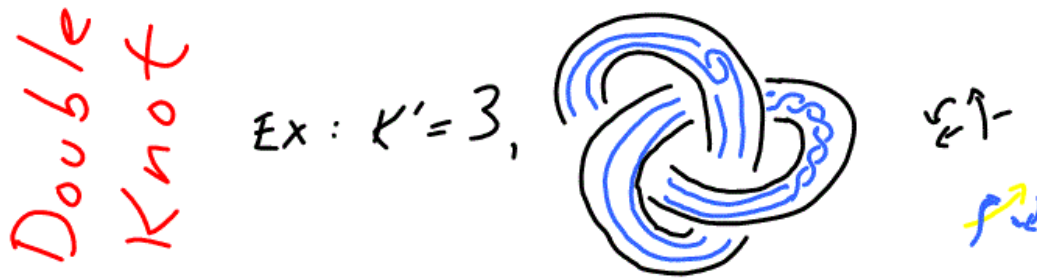
Let K' be a knot and let V' be a tubular nbhd of K' . Let $h: V \rightarrow V'$, $h(L) = J$. Then J is a *satellite link* with *companion knot* K' .

Ex: *Double knots*: $L =$

If companion $K' =$ the unknot, then satellite $J =$ twist knot.

Ex: $h(L) = \ell + m \rightarrow$

To uniquely define J , require h to be *faithful*: $h(S^1 \times 1) =$ preferred longitude.



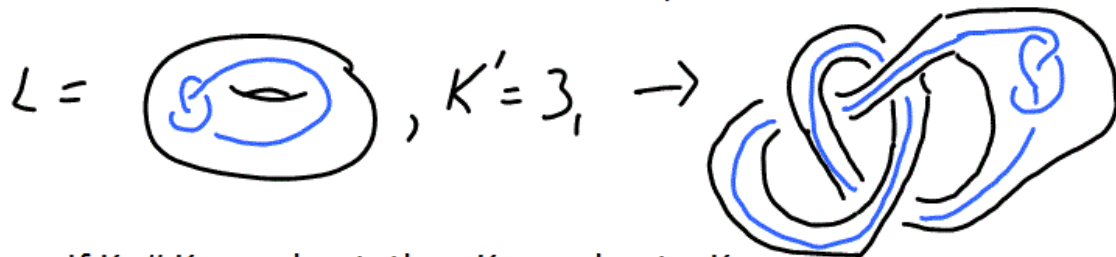
Ex: *Cable knot*: $L = (p, q)$ torus knot, $p \neq 0$, and the companion K' is non-trivial. If h is faithful, then J is called the (p, q) cable of K' .

If $p = \pm 1$, then $J =$

Thm 4D9: If K' is a companion of J , then $\pi_1(J)$ contains a subgroup isomorphic to $\pi_1(K')$

Cor 4D10: Any knot with non-trivial companion is non-trivial.

Note: $K_1 \# K_2$ is a satellite knot with companion K_1 .



Cor: If $K_1 \# K_2 = \text{unknot}$, then $K_1 = \text{unknot} = K_2$.

Define $K' \leq J$ if J is a satellite knot with companion K' .

Note \leq is reflexive and transitive and thus \leq gives a partial order of knot types.

Thm (Thurston 1978): A knot is either a (1) torus knot or (2) satellite knot or (3) a hyperbolic knot.

Minimal genus Seifert surfaces are not unique:

Satellite
Knot
w/ companion
 4_1

