Thursday, April 15, 2010

Def: $K \subset S^3 = \partial D^4$ is slice if $\exists$ disk $\Delta^2 \subset D^4 \exists \partial \Delta^2 = K \& \exists$ tubular neighborhood $\Delta^2 \times D^2 \exists (\Delta^2 \times D^2) \cap S^3 = K \times D^2$

Note: We can take $(\Delta^2)^c \subset IR^4$ where $K \subset IR^3 \times 0$.

$\Delta^2 \cup m(\Delta^2)$ is a smooth knotted $S^2$ in $S^4 \exists \partial [\Delta^2 \cup m(\Delta^2) \cap (IR^3 \times 0)] = K$

Suppose $K = (\text{knotted } S^2 \cap (IR^3 \times 0))$.

Then $IR^4 \cap \text{knotted } S^2 = \Delta^2$

$k$-manifold

$\partial \Delta^2 = K$

Def: $F^K \subset M^m$ is locally flat at $x \in F^K$ if $\exists$ closed neighborhood $N$ of $x \in M \cap (N, F^K \cap N) \cong (B^m, B^k)$

Thus, in slice definition, $\Delta^2$ is locally flat.

Proposition: $r: S^3 \to S^3$ orientation reversing homeomorphism

$\sigma_{F(M)}(r(K)) = -\sigma_M(K)$

Proposition 2: $\sigma_{M_1 \# M_2}(K_1 \# K_2) = \sigma_{M_1}(K_1) \# \sigma_{M_2}(K_2)$

Theorem 10: $K$ slice $\Rightarrow \sigma(K) = 0$.

Lemma 10: $K \# K^*$ slice

Corollary 12: $\sigma_{M_1}(K) = \sigma_{M_2}(K)$

(If $M_1$ & $M_2$ are 2 different Seifert surfaces)

Proof: $\sigma_{M_1}(K) = \sigma_{M_2}(K) = \sigma_{M_1}(K) + \sigma_{M_2}(r(K))$

$= \sigma_{M_1 \# r(M_2)}(K \# r(K)) = 0$

Section 8F: Concordance

Def: A concordance between $K_0, K_1 \subset S^3$ is a locally flat
The cylinder \( C \cong \mathbb{S}^1 \times [0, 1] \) embedded in \( \mathbb{S}^3 \times [0, 1] \) is
\[ \mathbb{S}^1 \times \{0\} = K_0 \times \{0\} \]
\[ \mathbb{S}^1 \times \{1\} = K_1 \times \{1\} \]

**Def:** \( K_0 \sim K_1 \iff K_0 \# K_1 \) are concordant.

**Lemma:** \( \sim \) is an equivalence relation

**Note:** Concordance preserves orientation

Note: Seifert Surface for \( K \)

Not Seifert Surface for \( O \)

Unlike for Seifert Surface for Links

**Note:** \( K \sim O \iff K \text{ slice} \)

**Equivalence Classes:** Let \( [T] = [K | K \sim J] \)
\[ [O_1] = [K | K \text{ slice}] \]

Let \( C_1 = \{ [K] | K \text{ oriented } \mathbb{S}^1 \text{ in } \mathbb{S}^3 \} \)

Define \( + : [T] + [K] = [T \# K] \)

**Theorem:** \( (C_1, +) \) is an abelian group.
- Well-defined
- Associative
- Identity = \([O_1]\)

- Abelian
- Inverses (given by concordance)

**Lemma:** If \( K \in [T] \), then \( \sigma(K^y) = \sigma(T) \)

\( \sigma \) is a homomorphism on \( C_1 \)

\( \sigma : C_1 \rightarrow \mathbb{Z} \) homomorphism