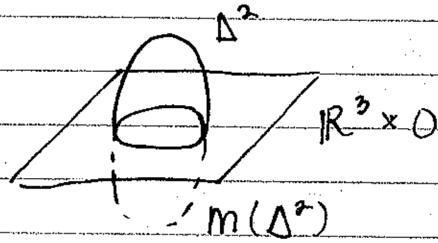
Thursday, April 15, 2010

Def: $K \subset S^3 = \partial D^4$ is slice if $\exists$ disk $\Delta^2 \subset D^4 \ni \partial \Delta^2 = K$ & $\exists$ tubular neighborhood $\Delta^2 \times D^2 \ni (\Delta^2 \times D^2) \cap S^3 = K \times D^2$

Note: We can take $(\Delta^2)^\circ \subset \mathbb{R}^4$ where $K \subset \mathbb{R}^3 \times 0$.



$\Delta^2 \cup m(\Delta^2)$ is a smooth knotted S^2 in $S^4 \ni$ $\partial[\Delta^2 \cup m(\Delta^2) \cap (\mathbb{R}^3 \times 0)] = K$

Suppose $K = (\text{Knotted } S^2 \cap (\mathbb{R}^3 \times 0))$.

Then $\mathbb{R}^4_+ \cap$ Knotted $S^2 = \Delta^2$
$= \text{disk}$
$\partial \Delta^2 = K$

k-manifold

Def: $F^k \subset M^m$ is locally flat at $x \in F^k$ if $\exists$ closed neighborhood N of x in $M \ni (N, F^k \cap N) \cong (B^m, B^k)$

Thus, in slice definition, Δ^2 is locally flat.

Proposition: $r: S^3 \to S^3$ orientation reversing homeomorphism
$\sigma_{r(M)}(r(K)) = -\sigma_M(K)$

Proposition 7: $\sigma_{M_1 \# M_2}(K_1 \# K_2) = \sigma_{M_1}(K) \# \sigma_{M_2}(K_2)$

$$\begin{bmatrix} V_1 & 0 \\ 0 & V_2 \end{bmatrix} \qquad \sigma_{M_1}(K) \downarrow V_1 \qquad \sigma_{M_2}(K_2) \downarrow V_2$$

Theorem 10: K slice $\Rightarrow \sigma(K) = 0$.

Lemma 10: $K \# K^*$ slice

Corollary 12: $\sigma_{M_1}(K) = \sigma_{M_2}(K)$
(so M_1 & M_2 are 2 different Seifert surfaces)

Proof: $\sigma_{M_1}(K) - \sigma_{M_2}(K) = \sigma_{M_1}(K) + \sigma_{M_2}(r(K))$
$= \sigma_{M_1 \# r(M_2)}(K \# r(K)) = 0$ ∎

Section 8F: Concordance

Def: A concordance between $K_0, K_1 \subset S^3$ is a locally flat cylinder $C \cong S^1 \times [0,1]$ embedded in $S^3 \times [0,1]$ ∋

$S^1 \times \{0\} = K_0 \times \{0\}$

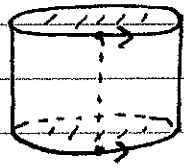
$S^1 \times \{1\} = K_1 \times \{1\}$

Def: $K_0 \sim K_1 \iff K_0$ & K_1 are concordant.

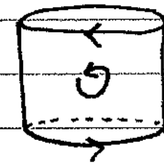
Lemma: $\sim$ is an equivalence relation

Note: concordance preserves orientation

Not Seifert surface for O_1

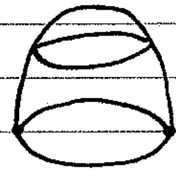


Unlike for Seifert surface for links

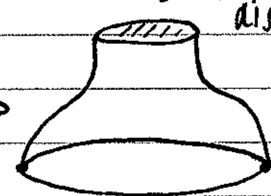


Note: $K \sim O_1 \iff K$ slice

$\Leftarrow$



$\Rightarrow$



Equivalence classes: Let $[J] = [K \mid K \sim J]$

$[O_1] = [K \mid K \text{ slice}]$

$K \# K^* \in [O_1]$

Let $\mathcal{C}_1 = \{[K] \mid K \text{ oriented } S^1 \text{ in } S^3\}$

Define $+$: $[J] + [K] = [J \# K]$

Theorem: $(\mathcal{C}_1, +)$ is an abelian group.

- well-defined ✓
- associative ✓
- identity $= [O_1]$ ✓
- abelian ✓
- inverses (given by concordance) ✓ $[K] + [K^*] = [O_1]$

Lemma: If $K \in [J]$, then $\sigma(K) = \sigma(J)$

↑ homomorphism on $\mathcal{C}_1$

$\sigma : \mathcal{C}_1 \longrightarrow 2\mathbb{Z}$ homomorphism