Tuesday, April 13, 2010

<u>Recall</u>: $H_1(\Sigma_K) = \langle m_1^0, \ldots, m_{2g}^0, \ldots, m_1^{K-1}, \ldots, m_{2g}^{K-1} \mid \vec{m}^i V^T = \vec{m}^{i+1} V \rangle$

$i = 0, \ldots, K-1 \mod K$

<u>Presentation Matrix</u>:

$$P = \begin{array}{c} \\ \text{gen}1 \\ \text{gen}2 \\ \vdots \\ \vdots \\ \text{gen}2gK \end{array} \begin{array}{c} \begin{array}{ccccc} \text{eqn}1 & \text{eqn}2 & \cdots & & \end{array} \\ \begin{bmatrix} V^T & -V & & & \\ & V^T & -V & & \\ & & \ddots & & \\ & & & \ddots & \\ -V & & & & V^T \end{bmatrix}_{2gK \times 2gK} \end{array}$$

$$\underset{\text{row operations}}{\sim} \begin{bmatrix} & & \ddots & & \\ & & & V & -V^T \\ -V^T & & & & V \\ V & -V^T & & & \\ & & \ddots & & \end{bmatrix}$$

<u>Note</u>: If K is odd, $P \sim$ skew symmetric matrix

$$\Rightarrow \quad P \sim \begin{bmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_{2g} \end{bmatrix}$$

Also, K odd $\Rightarrow H_1(\Sigma_K) \cong (\mathbb{Z}_{d_1} \times \cdots \times \mathbb{Z}_{d_{gK}})^2$

<u>Section 8E: Signature & Slice</u>

<u>Sidenote</u>: $H_1(\Sigma_2) \Rightarrow P = V + V^T$

<u>Note</u>: $V + V^T$ is a symmetric matrix $(a_{ij} = a_{ji})$

<u>Def</u>: $D \cong S$ if $\exists$ invertible $P \ni PSP^T = D$ (~~diagonal matrix~~)

① <u>Spectral Theorem</u>: If over field $F \ni 1+1 \neq 0$,

S symmetric $\Rightarrow S \cong D$ where D is diagonal

② <u>Sylvester's Theorem</u>: Over $\mathbb{R}$, if D, D' are diagonal matrices,

$D \cong D' \iff$ p = # of pos. entries of D = # of pos entries of D'

n = " "neg." " = " "neg" "

z = " "0" " = " "0" "

In fact, then the following is true:

$$D \cong D' \cong \begin{bmatrix} \underbrace{\begin{matrix} 1 & & \\ & \ddots & \\ & & 1 \end{matrix}}_{p} & & \\ & \underbrace{\begin{matrix} -1 & & \\ & \ddots & \\ & & -1 \end{matrix}}_{n} & \\ & & \underbrace{\begin{matrix} 0 & & \\ & \ddots & \\ & & 0 \end{matrix}}_{z} \end{bmatrix}$$

Def: The signature of S, denoted σ, is the number $p-n$.

Note: If S, T are symmetric, $S \cong T \Rightarrow \sigma(S) = \sigma(T)$

Suppose $\sigma(S) = \sigma(T)$, S, T same size,
$\det(S)\det(T) \neq 0 \Rightarrow S \cong T$.

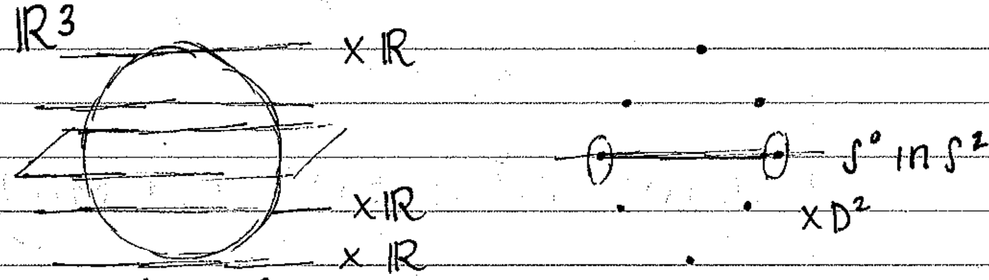
Def: $\sigma(K) = \sigma(V + V^T)$.

- $V + V^T = V^T + V$ so given Seifert surface M, choice of bicollar doesn't affect σ.
- We will show that σ does not depend on choice of M.

Slice Knots

Ambient space: $\mathbb{R}^3$

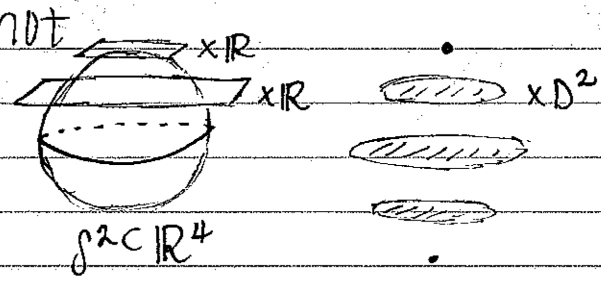
S^1:



S^1 in S^3

Def: K is a slice knot if $\exists S^n \subset \mathbb{R}^{n+2} \ni K = S^2 \cap H_{t_0}$ where $H_{t_0} = \{\vec{x} \in \mathbb{R}^{n+2} \mid x_0 = t_0\}$ ← smoothly embedded.

Ex: Unknot

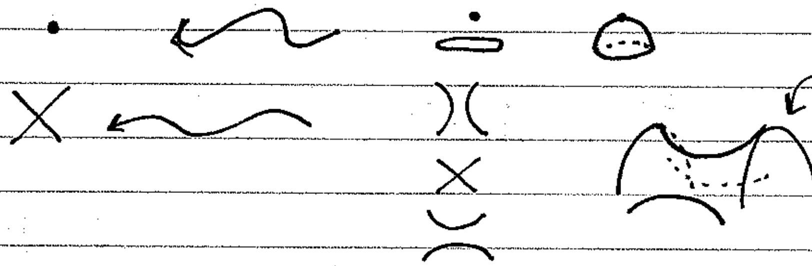


D_1 is a slice.

3_1 as twist knot

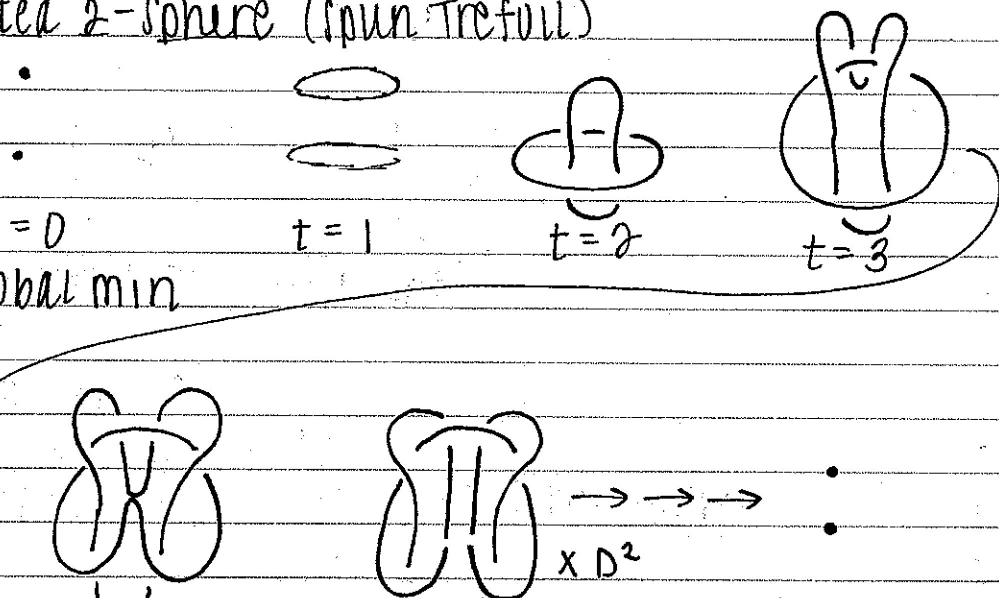
<u>Note</u>: A smooth knotted S^2 can be deformed $\ni$ all but a finite # of ~~crossings~~ cross-sections are ~~classical~~ classical links or $\varnothing$.

Exceptions:



saddle.

Knotted 2-<u>sphere</u> (spun Trefoil)



$t=0$ $t=1$ $t=2$ $t=3$

2 global min

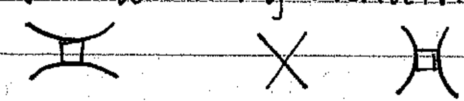
$t=4$ saddle point

$t=5$

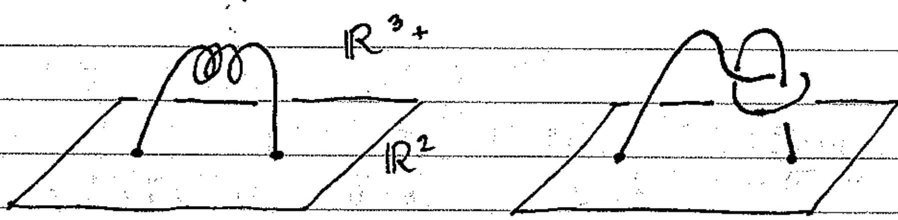
slice knot

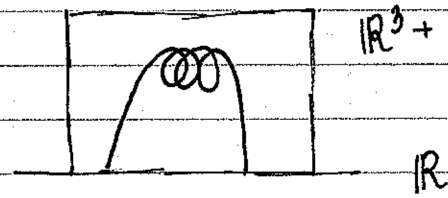
$t=10$

<u>Note</u>: Slice knots created by banding together unlinks.



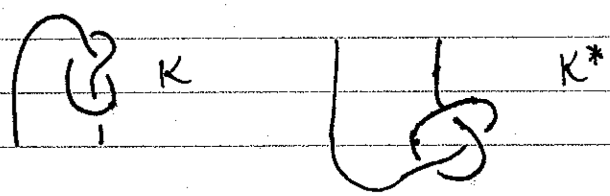
<u>Section 3J5: Spun Knots</u>





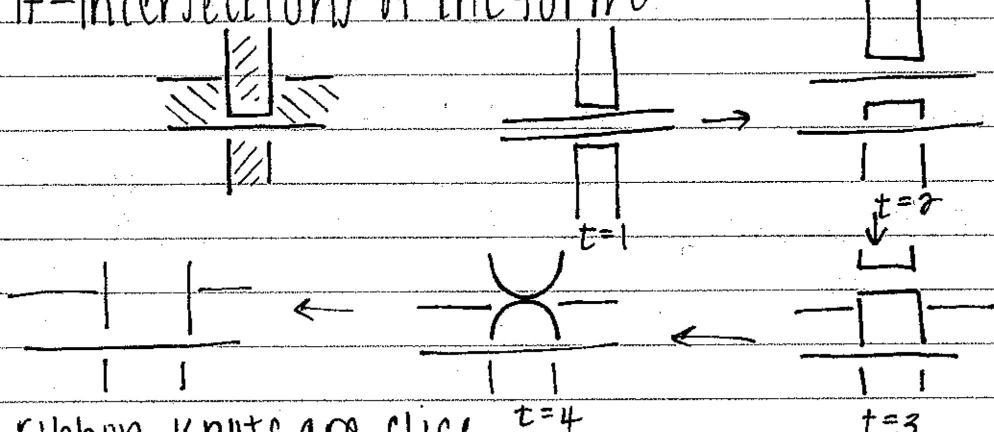
$(x_1, x_2, x_3, 0) \longrightarrow (x_1, x_2, x_3 \cos\theta, \sin\theta) \quad 0 \leq \theta \leq 2\pi$

$x_4 = 0 \Rightarrow \sin\theta = 0, \pi$
$(x_1, x_2, x_3 \cos(0), 0) \cup (x_1, x_2, x_3 \cos(\pi), 0)$
$\Rightarrow (x_1, x_2, x_3, 0) \cup (x_1, x_2, -x_3, 0)$



$\Rightarrow K \# K^*$ is slice knot

<u>Ribbon Knots</u>
$\partial M = K$, M is a disc.
M has self-intersections of the form



$\Rightarrow$ All ribbon knots are slice
<u>Conjecture</u>: All slice knots are ribbon knots

<u>Back to Signature (Section 8E)</u>
<u>Proposition 5</u>: $r: S^3 \to S^3$ orientation reversing homeomorphism
$\Rightarrow \sigma_{r(M)}(r(K)) = -\sigma_M(K)$ & $\sigma(K^*) = -\sigma(K)$

$\sigma(4_1) = \sigma(4_1^*)$ <u>Proof</u>: If M is Seifert surface for M, then r(M) is S.S. for r(K)
$\Rightarrow \sigma(4_1) = 0$ $\quad K \to M \to V$ $\quad \Rightarrow \sigma_{r(M)}(r(K)) = -\sigma_M(K)$
achiral $\Rightarrow \sigma = 0$ $\quad r(K) \to r(M) \to -V$ $\quad \sigma$ can detect mirror images