Final Exam May 13, 2009 SHOW ALL WORK
Math 28 Calculus III Either circle your answers or place on answer line.

Choose 8 out of the following 10 problems: **Clearly indicate which 8 problems you choose.** 7 of these problems will be worth 14 points. The 8th problem will be worth 7 points (2 points + 5 points extra credit). You may do all problems. If you do not correctly choose your top 8 problems, I may change your choices (with a small penalty) if it improves your grade.

My 8th choice (worth 7 points) is ______________

Do not grade the following 2 problem(s): ______________

Problems 1 - 4 or 5 will come from chapters 1 - 4. The problems will be "randomly" chosen, but extra weight will be given to the following sections:

Tangent (hyper-)planes/linear approximations
2.5 Chain rule
2.6 Directional derivative
3.3 Vector Field
4.2 Extrema
4.4 Lagrange Multiplier

The next 4-5 problems will come from chapters 5 - 7.

Problem 10.) Prove either (A) or (B). Clearly indicate your choice.

You may bring a 3 × 5 card to the exam. You may write on both sides of this card.

Scalar Line Integrals:

Let $\mathbf{x} : [a,b] \to \mathbf{R}^n$ be a C^1 path. $f : \mathbf{R}^n \to \mathbf{R}$, a scalar field.

Δs_k = length of kth segment of path

$$= \int_{t_{k-1}}^{t_k} ||\mathbf{x}'(t)||dt = ||\mathbf{x}'(t_k^{**})||(t_k - t_{k-1}) = ||\mathbf{x}'(t_k^{**})||\Delta t_k$$

for some $t_k^{**} \in [t_{k-1}, t_k]$

$$\int_{\mathbf{x}} f \ ds \sim \Sigma_{i=1}^n f(\mathbf{x}(t_k^*))\Delta s_k = \Sigma_{i=1}^n f(\mathbf{x}(t_k^*))||\mathbf{x}'(t_k^{**})||\Delta t_k$$

Thus $\int_{\mathbf{x}} f \ ds = \int_a^b f(\mathbf{x}(t))||\mathbf{x}'(t)||dt$

Scalar Surface integral

Suppose $S = X(D)$

$$\int\int_S f \ dS = lim_{\Delta A_k \to 0} \Sigma f(\mathbf{c}_k)\Delta A_k$$

$$= \int\int_D f(X(s,t))||T_s \times T_t||dsdt$$

$$= \int\int_D f(X(s,t))||N(s,t)||dsdt$$

Vector Line integrals:

Let $\mathbf{x} : [a, b] \to \mathbf{R}^n$ be a C^1 path. $F : \mathbf{R}^n \to \mathbf{R}^n$, a vector field.

$\mathbf{x}'(t_k^*) \sim \frac{\Delta \mathbf{x}_k}{\Delta t_k}$

$\int_{\mathbf{x}} F \cdot ds \sim \Sigma_{i=1}^n F(\mathbf{x}(t_k^*)) \cdot \Delta \mathbf{x}_k = \Sigma_{i=1}^n F(\mathbf{x}(t_k^*)) \cdot \mathbf{x}'(t_k^*) \Delta t_k$

Thus $\int_{\mathbf{x}} F \cdot ds = \int_a^b F(\mathbf{x}(t)) \cdot \mathbf{x}'(t) dt$

$=$ work done by F on particle as particle moves along path $\mathbf{x}$ and F represents a force field.

$= \int_{\mathbf{x}} F(\mathbf{x}(t)) \cdot T(t) ds$ where T is unit tangent to $\mathbf{x}$.

$=$ scalar line integral of the tangential component of F along path $\mathbf{x}$.

$=$ circulation of F along $\mathbf{x}$ when $\mathbf{x}$ is a closed curve.

If F is conservative and
if $A =$ the initial point of $\mathbf{x}$ and $B =$ the terminal point of $\mathbf{x}$,
then $\int_{\mathbf{x}} F \cdot ds = f(B) - f(A) =$ total change.

Note the scalar line integral $\int_C F \cdot \mathbf{n} ds =$ flux across C where $\mathbf{n}$ is the unit normal to C in the direction of interest.
6.2 Divergence Thm in the Plane $\int_{\partial S} F \cdot \mathbf{n} ds = \int \int_S [\nabla \cdot F] dA$

Vector Surface integral

$\int \int_X F \cdot dS = \int \int_X F(X(s, t)) \cdot N(s, t) ds dt = \int \int_X (F \cdot \mathbf{n}) dS$

$=$ flux of F across S

$= \dfrac{\text{volume of fluid flowing thru } S}{\text{unit time}}$

$=$ rate of fluid flow thru S

where F is a velocity vector field of a fluid.