Exam 1 March 5, 2008 — SHOW ALL WORK

Math 28 Calculus III — Either circle your answers or place on answer line.

[12] 1.) Let $f(x) = (x^2, ln(5-x^2), 2)$. Let $g(x, y, z) = xyz$. Use the chain rule to calculate $D(f \circ g)(1, 2, 3)$ and $D(g \circ f)(2)$

$D(f \circ g)(1, 2, 3) =$ ________________ $D(g \circ f)(2) =$ ________________

2.) Suppose $f(x, y) = ln(xy)$.

[10] a.) Find an equation for the tangent plane to the graph of f at the point $(2, \frac{1}{2}, 0)$

[4] b.) Approximate $f(1.8, 0.6) =$ ________________

[3] c.) A vector normal to this tangent plane is ________________

[4] 3a.) $proj_{(3,2)}(5, 9) =$ ________________

[5] 3b.) Find a unit vector perpendicular to the vectors (3, 2, 4) and (-1, 5, 0)

[27] 4.) Suppose the elevation of is given by $h(x, y) = 4x^2 - y^2$. Suppose you are at the point $(x, y) = (1, 2)$.

[3] a.) $\nabla h(1, 2) =$ ______________

[3] b.) What is the direction of steepest ascent? ______________

[3] c.) What is the rate of increase in the direction of steepest ascent? ______________

[3] d.) what is the direction of steepest descent? ______________

[3] e.) What is the rate of decrease in the direction of steepest descent? ______________

[3] f.) What is the direction where there is no change in elevation? ______________

[3] g.) What is the rate of increase if you travel in the direction (3, 4)? ______________

[4] h.) Graph several level curves of f (make sure to indicate the height c of each curve). Draw unit vectors in the direction of the gradient at (1, 4) and at (0, 2).

[2] i.) Identify the quadric surface: ______________

[9] 5.) Find the following limit if it exists. If it doesn't exist, state why you know it doesn't exist.

$$lim_{(x,y)\to(1,0)} = \frac{y(x+3)}{x+y-1}$$

[3] 6a.) State the definition of $lim_{\mathbf{x}\to\mathbf{a}} f(x) = \mathbf{L}$.

[7] 6b.) Suppose $f : \mathbf{R}^2 \to \mathbf{R}$, $f(x,y) = 2y$. Prove that the $lim_{(x,y)\to(3,4)} f(x,y) = 8$

7.) Circle T for True and F for False:

[3] a.) Suppose $f : \mathbf{R}^n \to \mathbf{R}$. If $\frac{\partial f}{\partial x_i}(\mathbf{a})$ exists for $i = 1, ..., n$, then f is differentiable at $\mathbf{a}$. T F

[3] b.) Suppose $f : \mathbf{R}^n \to \mathbf{R}$. If $\frac{\partial f}{\partial x_i}(\mathbf{a})$ exists for $i = 1, ..., n$, then $D_{\mathbf{v}}(f) = \nabla f \cdot \mathbf{v}$. T F

[3] c.) Suppose $f : \mathbf{R}^n \to \mathbf{R}$. If f is differentiable at $\mathbf{a}$, then $D_{\mathbf{v}}(f) = \nabla f \cdot \mathbf{v}$. T F

[3] d.) Suppose $f : \mathbf{R}^n \to \mathbf{R}$ is smooth. Then $\frac{\partial^2 f}{\partial x_1 \partial x_2} = \frac{\partial^2 f}{\partial x_2 \partial x_1}$ T F

[3] e.) Suppose $f : \mathbf{R}^n \to \mathbf{R}$ is differentiable. Then $\frac{\partial^2 f}{\partial x_1 \partial x_2} = \frac{\partial^2 f}{\partial x_2 \partial x_1}$ T F

[3] f.) If f is differentiable, then f is continuous. T F

[3] g.) If $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbf{R}^n$, then $\mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v}$. T F

[3] h.) If $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbf{R}^n$, then $\mathbf{v} \times \mathbf{w} = \mathbf{w} \times \mathbf{v}$. T F