Scalar Line Integrals:

Let $x : [a, b] \to \mathbf{R}^3$ be a $C^1$ path.

$$\Delta s_k = \text{length of kth segment of path}$$
$$= \int_{t_{k-1}}^{t_k} ||x'(t)||dt = ||x'(t_k^{**})||(t_k - t_{k-1}) = ||x'(t_k^{**})||\Delta t_k$$
$$\text{for some } t_k^{**} \in [t_{k-1}, t_k]$$

$$\int_x f\ ds \sim \Sigma_{i=1}^n f(x(t_k^*))\Delta s_k = \Sigma_{i=1}^n f(x(t_k^*))||x'(t_k^{**})||\Delta t_k$$
$$= \int_a^b f(x(t))||x'(t)||dt$$

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Vector Line integrals:

Calc 1 review: Suppose $f : \mathbf{R} \to \mathbf{R}$,

$\int_a^b f'(t)dt = f(b) - f(a)$.

$\int_a^b$(velocity) $dt =$ distance traveled.

$\int_a^b$(rate of change)$dt =$ total change.

Given $f'(t)$, can find $f(b) - f(a)$.

Given velocity, can find distance traveled.

Given rate of change, can find total change.

Calc III: Suppose $f : \mathbf{R}^2 \to \mathbf{R}$,

Given $\nabla f = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y})$, find $f(\mathbf{p}) - f(\mathbf{q})$.