$f(x,y) = ln(\frac{xy}{3})$.

$\nabla f(x,y) = Df(x,y) = (\frac{1}{x}, \frac{1}{y})$

Hessian of $f = Hf(x,y) = \begin{pmatrix} \frac{\partial[\nabla f(x,y)]}{\partial x} \\ \frac{\partial[\nabla f(x,y)]}{\partial y} \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 f}{\partial^2 x} & \frac{\partial^2 f}{\partial y \partial x} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial^2 y} \end{pmatrix}$

Thus $Hf(x,y) = \begin{pmatrix} -x^{-2} & 0 \\ 0 & -y^{-2} \end{pmatrix}$

Let $\mathbf{a} = (3,1)$. Then $f(3,1) = ln(\frac{(3)(1)}{3}) = ln(1) = 0$.

$\nabla f(3,1) = Df(3,1) = (\frac{1}{3}, 1)$

$Hf(3,1) = \begin{pmatrix} -3^{-2} & 0 \\ 0 & -1^{-2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{9} & 0 \\ 0 & -1 \end{pmatrix}$

First order approximation of f near $\mathbf{a} = (3,1)$:

Tangent plane to $f(x,y) = ln(x,y)$ at $\mathbf{a} = (3,1)$ is

$$p_1(x,y) = f(\mathbf{a}) + Df(\mathbf{a})(\mathbf{x} - \mathbf{a})$$

$$p_1(x,y) = f(3,1) + Df(3,1)\begin{pmatrix} x-3 \\ y-1 \end{pmatrix} = 0 + (\tfrac{1}{3}, 1)\begin{pmatrix} x-3 \\ y-1 \end{pmatrix}$$

$$= \tfrac{1}{3}(x-3) + y - 1 = \tfrac{1}{3}x - 1 + y - 1.$$

Thus $p_1(x,y) = \frac{1}{3}x + y - 2$.

Second order approximation of f near $\mathbf{a} = (3,1)$:

$$p_2(x,y) = f(\mathbf{a}) + Df(\mathbf{a})(\mathbf{x}-\mathbf{a}) + \tfrac{1}{2}(\mathbf{x}-\mathbf{a})^T Hf(\mathbf{a})(\mathbf{x}-\mathbf{a})$$

$$p_2(x,y) = \tfrac{1}{3}x + y - 2 + \tfrac{1}{2}(x-3, y-1)\begin{pmatrix} -\frac{1}{9} & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x-3 \\ y-1 \end{pmatrix}$$

$$p_2(x,y) = \tfrac{1}{3}x + y - 2 + \tfrac{1}{2}(x-3, y-1)\begin{pmatrix} -\frac{1}{9}(x-3) \\ -(y-1) \end{pmatrix}$$

$$p_2(x,y) = \tfrac{1}{3}x + y - 2 - \tfrac{1}{18}(x-3)^2 - \tfrac{1}{2}(y-1)^2$$