2.4 Higher order derivatives:

\[
\frac{\partial^2 f}{\partial x_{i_1} \partial x_{i_2}} = \frac{\partial}{\partial x_{i_1}}(\frac{\partial}{\partial x_{i_2}}(f))
\]

Ex: Let \( f(x, y, z) = x^2 \ln(yz) \)

\[
\begin{align*}
\frac{\partial f}{\partial x} &= \quad \quad \frac{\partial f}{\partial y} = \quad \quad \frac{\partial f}{\partial z} = \\
\frac{\partial^2 f}{\partial x^2} &= \quad \quad \frac{\partial^2 f}{\partial x \partial y} = \quad \quad \frac{\partial^2 f}{\partial z \partial y} = \\
\frac{\partial^3 f}{\partial x^3} &= \quad \quad \frac{\partial^3 f}{\partial x^2 \partial z} = \quad \quad \frac{\partial^3 f}{\partial x \partial y \partial z} =
\end{align*}
\]

Defn: Let \( V \) be a nonempty open subset of \( \mathbb{R}^n \), \( f : V \to \mathbb{R}^m \), \( p \in \mathbb{N} \).

i.) \( f \) is \( C^p \) on \( V \) if each partial derivative of order \( k \leq p \) exists and is continuous on \( V \).

ii.) \( f \) is \( C^\infty \) on \( V \) if \( f \) is \( C^p \) on \( V \) for all \( p \in \mathbb{N} \) (\( f \) is smooth).

Ex: \( g(x, y) = (x + y, x) \)

Cor 1.7 If \( f \in C^r \) on \( U \), then 

\[
\frac{\partial^k f}{\partial x_{i_1} \partial x_{i_2} \ldots \partial x_{i_k}} = \frac{\partial^k f}{\partial x_{j_1} \partial x_{j_2} \ldots \partial x_{j_k}}
\]

where \((j_1, j_2, \ldots, j_k)\) is a permutation of \((i_1, i_2, \ldots, i_k)\)