

[10] 1a.) Use a linear approximation (or differentials) to estimate  $\ln(0.97)$

Answer 1a.) \_\_\_\_\_

[2] 1b.) Is the answer to 1a an over-estimate or an under-estimate? \_\_\_\_\_

[2] 1c.) For the problem in 1a,

$dx =$  \_\_\_\_\_,  $dy =$  \_\_\_\_\_,  $\Delta x =$  \_\_\_\_\_,  $\Delta y =$  \_\_\_\_\_.

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[2] 2a.) The linearization of  $f(x) = \sin(x)$  at  $x = 0$  is \_\_\_\_\_

[2] 2b.) The linearization of  $f(x) = \cos(x)$  at  $x = 0$  is \_\_\_\_\_

[2] 2c.) The linearization of  $f(x) = \sin(x)$  at  $x = \frac{\pi}{2}$  is \_\_\_\_\_

[2] 2d.) The linearization of  $f(x) = 2x + 1$  at  $x = 0$  is \_\_\_\_\_

[2] 2e.) Use the above linearizations to estimate the following:

$$\sin(0.1) \sim \text{_____}, \quad \sin\left(\frac{3}{2}\right) \sim \text{_____}$$

[10] 3.) Recall that radioactive substances decay at a rate proportional to the remaining mass. The half-life of polonium-218 is 3 minutes. Suppose a sample originally has a mass of 400g. SIMPLIFY your answers to the following:

a.) A formula for the mass remaining after  $t$  minutes is \_\_\_\_\_

b.) The mass remaining after 6 minutes is \_\_\_\_\_

c.) When is the mass reduced to 10g? \_\_\_\_\_

[15] 4.) Find the derivative of  $x \ln(\sqrt{3\sin(x^2) - e^x + 1})$ .  
Circle your answer. You do NOT need to simplify.

[15] 5.) A plane flies horizontally at an altitude of 10km and passes directly over a tracking telescope on the ground. When the angle of elevation is  $\pi/3$  (with respect to the tracking telescope, after it has passed over the tracking telescope), this angle is decreasing at a rate of  $\pi/4$  rad/min. How fast is the plane traveling at that time. (Hint: you can use a right triangle).

Answer) \_\_\_\_\_

6.) Find the following for  $f(x) = x^3 - 8x^2 + 16x = x(x - 4)^2$  (if they exist; if they don't exist, state so). Use this information to graph  $f$ .

Note  $f'(x) = 3x^2 - 16x + 16 = (x - 4)(3x - 4)$  and  $f''(x) = 6x - 16$

[1.5] 6a.) critical numbers: \_\_\_\_\_

[1.5] 6b.) local maximum(s) occur at  $x =$  \_\_\_\_\_

[1.5] 6c.) local minimum(s) occur at  $x =$  \_\_\_\_\_

[1.5] 6d.) The global maximum of  $f$  on the interval  $[0, 5]$  is \_\_\_\_\_ and occurs at

$x =$  \_\_\_\_\_

[1.5] 6e.) The global minimum of  $f$  on the interval  $[0, 5]$  is \_\_\_\_\_ and occurs at

$x =$  \_\_\_\_\_

[1.5] 6f.) Inflection point(s) occur at  $x =$  \_\_\_\_\_

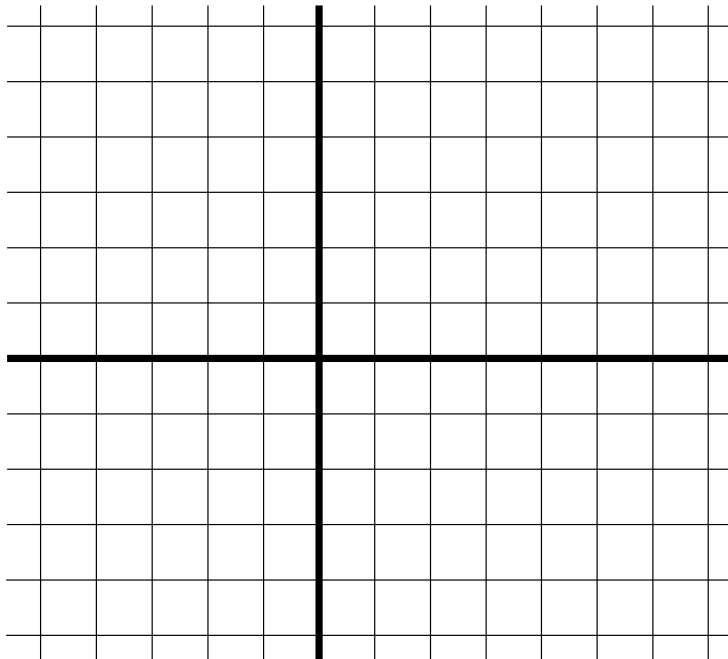
[1.5] 6g.)  $f$  increasing on the intervals \_\_\_\_\_

[1.5] 6h.)  $f$  decreasing on the intervals \_\_\_\_\_

[1.5] 6i.)  $f$  is concave up on the intervals \_\_\_\_\_

[1.5] 6j.)  $f$  is concave down on the intervals \_\_\_\_\_

[5] 6k.) Graph  $f$



[16] 7.) Circle T for true and F for false. If the statement is false, give a counter-example.

7a.) If  $f'(x) > 0$  on an interval, then  $f$  is increasing on that interval. T

Counter-example: None.

7b.) If  $f$  is increasing on an interval, then  $f'(x) > 0$  on that interval. F

Counter-example:  $f(x) = x^3$  is an increasing function on  $(-\infty, \infty)$ , but  $f'(0) = 0$ .

7c.) If  $f'(c) = 0$ , then  $f$  has a local maximum or local minimum at  $c$ . T F

Counter-example:

7d.) If  $f$  has a local maximum or local minimum at  $c$ , then  $f'(c) = 0$ . T F

Counter-example:

7e.) If  $f$  has a local maximum or local minimum at  $c$  and if  $f'(c)$  exists, then  $f'(c) = 0$ . T F

Counter-example:

7f.) Suppose  $f''$  is continuous near  $x$  and  $f'(c) = 0$ . If  $f''(c) > 0$ , then  $f$  has a local minimum at  $c$ . T F

Counter-example:

7g.) Suppose  $f''$  is continuous near  $x$  and  $f'(c) = 0$ . If  $f$  has a local minimum at  $c$ , then  $f''(c) > 0$ . T F

Counter-example:

7h.) If  $f$  is continuous on  $(a, b)$ , then  $f$  attains an absolute maximum value  $f(c)$  at some number  $c$  in  $(a, b)$ . T F

Counter-example: