Exam 1 Feb 22, 2007 SHOW ALL WORK
Math 25 Calculus I Either circle your answers or place on answer line.

Find the following derivatives (you do not need to simplify):

[14] 1.) $\frac{d}{dx}[\frac{x^2+3\sqrt{x}+x}{2x^4-5}]$

$$= \frac{(2x^4-5)\frac{d}{dx}(x^2+3x^{1/2}+x) - (x^2+3x^{1/2}+x)\frac{d}{dx}(2x^4-5)}{(2x^4-5)^2}$$

$$= \frac{(2x^4-5)(2x+\frac{3}{2}x^{-1/2}+1) - (x^2+3x^{1/2}+x)(8x^3)}{(2x^4-5)^2}$$

Answer 1.) ______________________________

[14] 2.) $\frac{d}{dx}[2xe^x + 3\sqrt{x^5} - \frac{1}{x}]$

$$\frac{d}{dx}(2xe^x) + \frac{d}{dx}(3x^{5/2}) - \frac{d}{dx}(x^{-1})$$

$$= 2e^x + 2xe^x + \frac{15}{2}x^{3/2} + x^{-2}$$

Answer 2.) ______________________________

3.) Calculate the appropriate limits in order to find the equations of all vertical and horizontal asymptotes for $f(x) = \frac{\sqrt{x^2+1}}{2(x-3)}$. Show ALL steps.

horizontal asymptotes :

$$\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{2(x-3)} = \lim_{x\to\infty} \frac{\sqrt{x^2(1+\frac{1}{x^2})}}{2x(1-\frac{3}{x})} = \lim_{x\to\infty} \frac{|x|\sqrt{1+\frac{1}{x^2}}}{2x(1-\frac{3}{x})} \quad \cdots (*)$$

$$= \lim_{x\to\infty} \frac{x\cdot\sqrt{1+\frac{1}{x^2}}}{2x(1-\frac{3}{x})} = \lim_{x\to\infty} \frac{\sqrt{1+\frac{1}{x^2}}}{2(1-\frac{3}{x})} = \frac{1}{2}$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{2(x-3)} \overset{\text{by } (*)}{=\!=} \lim_{x\to-\infty} \frac{|x|\sqrt{1+\frac{1}{x^2}}}{2x(1-\frac{3}{x})} = \lim_{x\to-\infty} \frac{-x\cdot\sqrt{1+\frac{1}{x^2}}}{2x(1-\frac{3}{x})}$$

$$= \lim_{x\to-\infty} \frac{-\sqrt{1+\frac{1}{x^2}}}{2(1-\frac{3}{x})} = -\frac{1}{2}$$

[12] horizontal asymptotes) $y=\frac{1}{2}$ and $y=-\frac{1}{2}$

vertical asymptote.

If $x=a$ is a vertical asymptote, then $\lim_{x\to a} f(x) = \infty$ or $-\infty$ or both

So $\frac{\sqrt{x^2+1}}{2(x-3)} = \infty$ or $-\infty$ as $x\to a$.

Thus $x-3\to 0$, i.e. $x\to 3$

$$\lim_{x\to 3^+} \frac{\sqrt{x^2+1}}{2(x-3)} = +\infty, \quad \lim_{x\to 3^-} \frac{\sqrt{x^2+1}}{2(x-3)} = -\infty$$

[10] vertical asymptotes) $x=3$.

[10] 4a.) Find the derivative of $f(x) = 2x + 3$ by using the definition of derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h)+3 - [2x+3]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x} + 2h + \cancel{3} - \cancel{2x} - \cancel{3}}{h}$$

$$= \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}} = \lim_{h \to 0} 2 = 2$$

$f'(x) =$ 2

[3] 4b.) Find the **equation** of the tangent line to the curve $f(x) = 2x + 3$ when $x = 1$.

$$y = 2x + 3$$

[10] 5.) Express the given quantity as a single logarithm.:

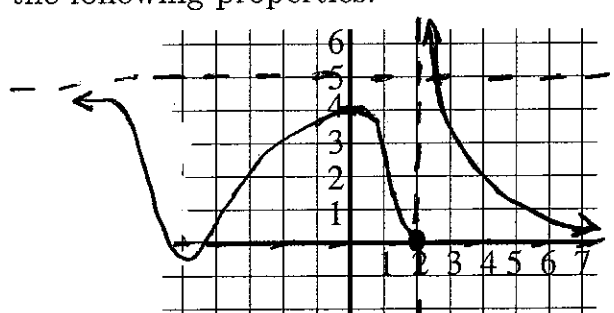
$a\ ln(x) + b\ ln(y) - c\ ln(z) - d\ ln(1) =$ $\ln\left(\frac{x^a y^b}{z^c}\right)$

$$\ln x^a + \ln y^b - \ln z^c - 0$$

$$\ln\left(\frac{x^a y^b}{z^c}\right)$$

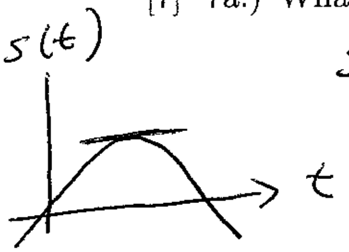
[7] 6.) Sketch the graph of a function with the following properties:

$lim_{x \to 2^+} f(x) = +\infty$,
$lim_{x \to +\infty} f(x) = 0$,
$lim_{x \to -\infty} f(x) = 5$
$f'(-3) = 1, f'(0) = 0, f'(1) = -4$



7.) If a ball is thrown vertically upward with a velocity of 16 ft/sec, then its height (in feet) is given by $s(t) = 16t - 16t^2$.

[7] 7a.) What is the maximum height reached by the ball? $\underline{4 \text{ ft}}$



$$s'(t) = 16 - 32t = 0$$
$$16 = 32t$$
$$\Rightarrow t = \frac{16}{32} = \frac{1}{2}$$

$$s\left(\tfrac{1}{2}\right) = 16\left(\tfrac{1}{2}\right) - 16\left(\tfrac{1}{2}\right)^2$$
$$= 8 - 4 = 4$$

[3] 7b.) Find a point $(t_0, s(t_0))$ at which the slope of the tangent line to the curve $s(t) = 16t - 16t^2$ is equal to 0: $(t_0, s(t_0)) = \underline{(1/2,\ 4)}$

[10] **Choose either problem 8 or 9. You may do both problems for up to 4 points extra credit.**

8.) Let $f : R \to R$, $f(x) = (x-3)^2$.

8a.) Is f 1:1? $\underline{NO}$. If f is not 1:1, prove it.

$$f(0) = 9 = f(6)$$

b.) Is f onto? $\underline{NO}$. If f is not onto, prove it.

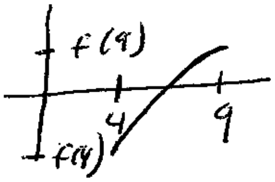
$(x-3)^2 \geq 0 \Rightarrow -1$ is not in range $= [0, \infty)$
but -1 is in codomain $= \mathbb{R}$
Thus range $\neq$ codomain

9a.) State the Intermediate Value Theorem.

Suppose f in continuous on $[a,b]$, $f(a) \neq f(b)$
and N is between $f(a)$ and $f(b)$
Then there exists $c \in (a,b)$ such that $f(c) = N$

9b.) Use the Intermediate Value Theorem to show that $\sqrt{x} - \frac{5}{2} = 0$ has a root between 4 and 9.

$f(x) = \sqrt{x} - 5/2$ is continuous on $[4, 9]$



$f(4) = -1/2$, $f(9) = 1/2 \Rightarrow f(4) \neq f(9)$

$-\frac{1}{2} < 0 < 1/2$

$\Rightarrow$ there exists $c \in (4, 9)$ st $f(c) = 0$

$\Rightarrow f(c) = \sqrt{c} - 5/2 = 0$. Thus $\sqrt{x} - 5/2$ has a root btwn 4 & 9