Exam 2 April 5, 2005 SHOW ALL WORK
Math 25 Calculus I Either circle your answers or place on answer line.

[10] 1a.) Find the linearization of $f(x) = x^2 + 3x$ at $x = 0$.

$f'(x) = 2x + 3$

$f'(0) = 3$, $f(0) = 0$

So $y - f(0) = f'(0)(x - 0)$

$y - 0 = 3x$

$y = 3x$

Answer 1a.) $y = 3x$

[6] 1b.) Use this linearization to approximate $f(0.1)$.

$f(0.1) \sim 3 \times 0.1 = 0.3$

Answer 1b.) 0.3

[2] 1c.) $f''(0) =$ 2 $f''(x) = 2$

concave up ⟶

[2] 1d.) Is the answer to 1b an over-estimate or an under-estimate? under-estimate 2

[2] 1e.) In the example in 1b, $\Delta x =$ 0.1, $\Delta y =$ 0.31, $dx =$ 0.1, $dy =$ 0.3;

$\Delta x = 0.1 - 0$

$\Delta y = f(0.1) - f(0)$ ⟶ (bc) $\Delta y = [(0.1)^2 + 3 \cdot (0.1)] - 0$

$= 0.01 + 0.3$

$= 0.31$

(bc) $\frac{dy}{dx} = 3$ at $x = 0$

$dy = 3 \cdot dx = 3(0.1) = 0.3$

2.) Find y'.

[15] 2a.) $xy = 4(x+y)^{\frac{3}{2}}$, then $y' = \dfrac{6\sqrt{x+y} - y}{x - 6\sqrt{x+y}}$

$$y + xy' = 4 \times \frac{3}{2} \cdot (x+y)^{\frac{1}{2}} \cdot (1+y')$$

$$y + xy' = 6(x+y)^{\frac{1}{2}}(1+y')$$

$$y + xy' = 6\sqrt{x+y} + 6\sqrt{x+y} \cdot y'$$

$$xy' - 6\sqrt{x+y} \cdot y' = 6\sqrt{x+y} - y$$

$$(x - 6\sqrt{x+y})\, y' = 6\sqrt{x+y} - y$$

$$y' = \frac{6\sqrt{x+y} - y}{x - 6\sqrt{x+y}}$$

[15] 2b.) $y = x^{\cos(x)}$, then $y' = \left(-\sin x \cdot \ln x + \dfrac{\cos x}{x}\right) \cdot x^{\cos x}$

$$\ln y = \cos x \cdot \ln x$$

$$\frac{1}{y} \cdot y' = (\cos x)' \cdot \ln x + \cos x \cdot (\ln x)'$$

$$\frac{1}{y} \cdot y' = -\sin x \cdot \ln x + \cos x \cdot \frac{1}{x}$$

$$y' = \left(-\sin x \cdot \ln x + \frac{\cos x}{x}\right) \cdot y$$

$$y' = \left(-\sin x \cdot \ln x + \frac{\cos x}{x}\right) \cdot x^{\cos x}$$

[4] 3a.) State the extreme value theorem.

If f is continuous on $[a, b]$,
then there exists $c, d \in [a, b]$ such that
$f(c)$ is the absolute maximum of f on $[a, b]$
(ie $f(c) \geq f(x)$ for all $x \in [a, b]$)
and $f(d)$ is the absolute minimum of f on $[a, b]$
(i.e. $f(d) \leq f(x)$ for all $x \in [a, b]$).

[8] 3b.) Use calculus to find the absolute maximum and absolute minimum values of $f(x) = x^3 - 3x^2 + 1$ on $[-2, 3]$.

$f'(x) = 3x^2 - 6x = 3x(x-2)$

$f'(x) = 0$ at $x = 0$ and $x = 2$

$f(-2) = (-2)^3 - 3(-2)^2 + 1 = -8 - 12 + 1 = -19$ ← the smallest

$f(0) = 1$ ←

$f(2) = 2^3 - 3 \cdot 2^2 + 1 = 8 - 12 + 1 = -3$ the biggest

$f(3) = 3^3 - 3 \cdot 3^2 + 1 = 1$ ←

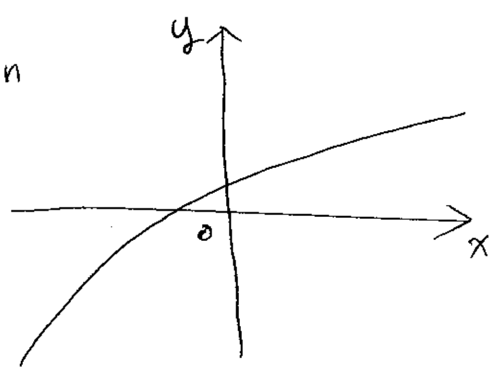
The absolute maximum is 1 and occurs at $x =$ 0 and 3

The absolute minimum is -19 and occurs at $x =$ -2

[5] 4.) Sketch the graph of a function whose first derivative is always positive and whose second derivative is always negative.

$f'(x) > 0 \rightarrow$ increasing

$f''(x) < 0 \rightarrow$ concave down

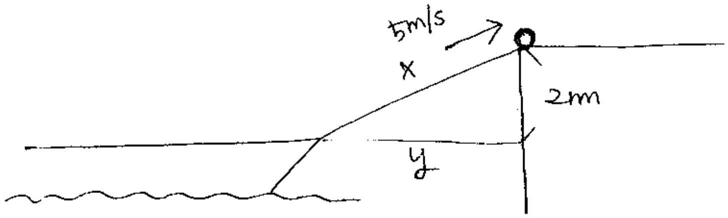


[15] 5.) Choose one of the following (clearly indicate your choice).

5a.) A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 2m higher than the bow of the boat. If the rope is pulled in at a rate of 5m/sec, how fast is the boat approaching the dock when it is 10m from the dock.

5b.) A particle is moving along the curve $y = x^2 - 1$. As the particle passes through the point (2, 3), it's x-coordinate increases at a rate of 6m/sec. How fast is the distance from the particle to the origin changing at this instant?

5a).



$\frac{dx}{dt} = -5 \text{ m/s}$

$\frac{dy}{dt} = ?$ at $y = 10$.

$$x^2 = y^2 + 2^2$$

$$2x \cdot \frac{dx}{dt} = 2y \cdot \frac{dy}{dt}.$$

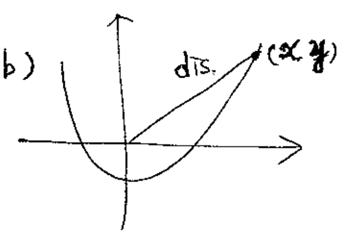
When $y = 10$, $x = \sqrt{10^2 + 2^2} = \sqrt{104}$

So

$$2 \cdot \sqrt{104} \cdot (-5) = 2 \cdot 10 \cdot \frac{dy}{dt} \Rightarrow \frac{dy}{dt} = -\frac{\sqrt{104}}{2}$$

Thus the boat is approaching at the rate of $\frac{\sqrt{104}}{2}$ m/s.

5b)



$\frac{dx}{dt} = 6$ m/s.

distance $l = \sqrt{x^2 + y^2}$. $\frac{dl}{dt} = ?$ when $x = 2, y = 3$.

$$l^2 = x^2 + y^2.$$

$$2l \frac{dl}{dt} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}.$$

$$l \frac{dl}{dt} = x \cdot \frac{dx}{dt} + y \cdot \frac{dy}{dt}.$$

2 6 3

Since $y = x^2 - 1$

$\frac{dy}{dt} = 2x \cdot \frac{dx}{dt} = 2 \cdot 2 \cdot 6 = 24$

$l = \sqrt{2^2 + 3^2}$
$= \sqrt{4+9}$
$= \sqrt{13}$

So $\sqrt{13} \cdot \frac{dl}{dt} = 2 \cdot 6 + 3 \cdot 24 = 12 + 72 = 84$.

Answer 5.) $\frac{dl}{dt} = \frac{84}{\sqrt{13}}$ m/s

6.) Find the following for $f(x) = \frac{x}{(x-1)^2}$ (if they exist; if they don't exist, state so). Use this information to graph f.

Note $f'(x) = \frac{-(x+1)}{(x-1)^3}$ and $f''(x) = \frac{2(x+2)}{(x-1)^4}$

[1] 6a.) critical numbers: $x = -1$ (where $f' = 0$ or f' : DNE in the domain)

[1.5] 6b.) local maximum(s) occur at $x =$ DNE

[1.5] 6c.) local minimum(s) occur at $x =$ -1

[1.5] 6d.) The global maximum of f on the interval [0, 5] is DNE and occurs at $x =$ DNE

[1.5] 6e.) The global minimum of f on the interval [0, 5] is 0 and occurs at $x =$ 0

[1.5] 6f.) Inflection point(s) occur at $x =$ -2

[1] 6g.) f increasing on the intervals $(-1, 1)$

[1] 6h.) f decreasing on the intervals $(-\infty, -1) \cup (1, \infty)$

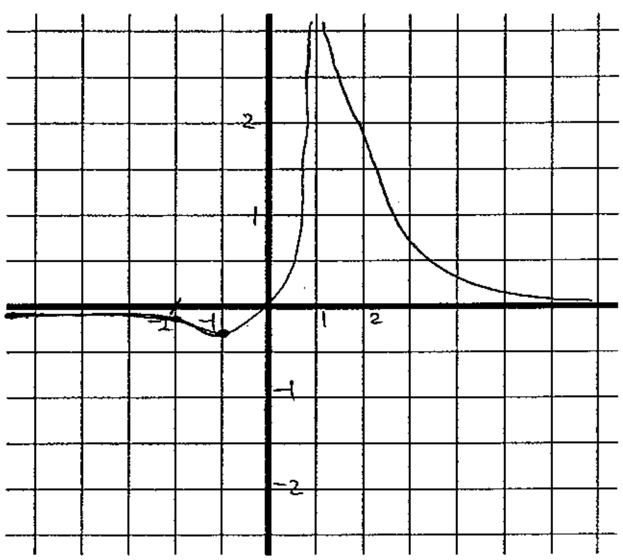
[1.5] 6i.) f is concave up on the intervals $(-2, 1) \cup (1, \infty)$

[1.5] 6j.) f is concave down on the intervals $(-\infty, -2)$

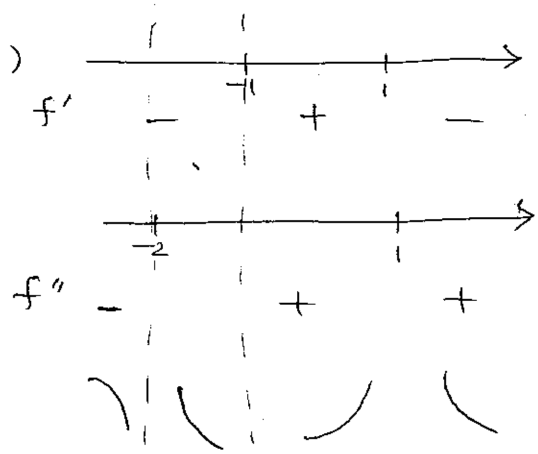
[1.5] 6k.) Equation(s) of vertical asymptote(s) $x = 1$

[2] 6l.) Equation(s) of horizontal asymptote(s) $y = 0$

[4] 6m.) Graph f



6b.)



$f(-2) = \frac{-2}{9}$, $f(0) = 0$

$f(-1) = \frac{-1}{4}$

$\lim_{x \to 1} f(x) = \infty$

$\lim_{x \to -\infty} \frac{x}{(x-1)^2} \stackrel{L}{=} \lim_{x \to -\infty} \frac{1}{2(x-1)} = 0$

Similarly $\lim_{x \to \infty} \frac{x}{(x-1)^2} = 0$