Exam 2 April 13, 2006 SHOW ALL WORK

Math 25 Calculus I Either circle your answers or place on answer line.

[14] 1.) Given $y = (x^2+1)^x$, find y'. Simplify your answer.

$$(x^2+1)^x = e^{\ln(x^2+1)^x} = e^{x\ln(x^2+1)}$$

$$y' = e^{x\ln(x^2+1)}\left[x\left(\frac{1}{x^2+1}\right)(2x) + \ln(x^2+1)\right]$$

$$= (x^2+1)^x\left[\frac{2x^2}{x^2+1} + \ln(x^2+1)\right]$$

Answer 1.) $(x^2+1)^{x-1}(2x^2) + (x^2+1)^x \ln(x^2+1)$

[13] 2.) Given $yx^2 + 10 = y^3$, find y''. You do NOT need to simplify your answer and you can leave your answer in terms of x and y (and only in terms of x and y, y' should not appear in your final answer).

$$y(2x) + y'(x^2) = 3y^2y'$$

$$\Rightarrow 2xy = (3y^2-x^2)y' \Rightarrow y' = \frac{2xy}{(3y^2-x^2)}$$

$$y'' = \frac{(2xy'+2y)(3y^2-x^2) - 2xy(3y^2y'-2x)}{(3y^2-x^2)^2}$$

Answer 2) $y'' = \dfrac{\left(\frac{4x^2y}{3y^2-x^2}+2y\right)(3y^2-x^2) - 2xy\left(\frac{6xy^3}{3y^2-x^2}-2x\right)}{(3y^2-x^2)^2}$

[14] 3.) Calculate the following limit. Show all steps.

$\lim_{x\to 0^+} x ln(x)$ = 0

"$0 \cdot \infty$"

$$\lim_{x\to 0^+} x \ln x = \lim_{x\to 0^+} \underbrace{\frac{\ln x}{x^{-1}}}_{``\frac{\infty}{\infty}"} \underset{L'H}{=} \lim_{x\to 0^+} \left(\frac{x^{-1}}{-x^{-2}}\right)$$

$$= \lim_{x\to 0^+} (-x) = 0$$

[5] 4a.) State the Mean Value Theorem

If f is continuous on $[a,b]$
and f is differentiable on (a,b)
then there exists $c \in (a,b)$ such that
$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

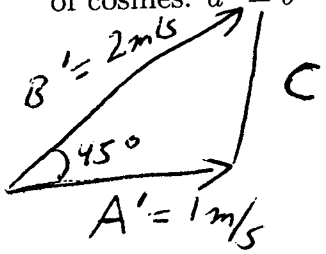
[8] 4b.) Use the Mean Value Theorem (or Rolle's theorem) to show $f(x) = \ln(x) + x$ is one-to-one [Hint: recall f is one-to-one if $f(a) = f(b)$ implies $a = b$. Assume $f(a) = f(b)$ and show $a = b$ WHEN a and b are in the domain of f].

Suppose $f(a) = f(b)$ and
a, b are in the domain of $f = (0,\infty)$
Suppose $a < b$
Note f is cont on $[a,b]$ & diff on (a,b)
Hence by the MVT there exists $c \in (a,b)$
such that $f'(c) = \frac{f(b)-f(a)}{b-a} = 0$

$f(x) = \ln x + x \Rightarrow f'(x) = \frac{1}{x} + 1 \Rightarrow f'(c) = \frac{1}{c} + 1$

But $c \in (0,\infty) \Rightarrow f'(c) = \frac{1}{c} + 1 > 0$, contradicting $f'(c) = 0$

[13] 5.) Two people start at the same point, say the origin. Person A walks east at a constant rate of 1m/s. Person B walks northeast (45 degrees north of east) at 2m/s. What is the rate of change in the distance between person A and person B after 20 seconds [law of cosines: $a^2 = b^2 + c^2 - 2bc\cos(\alpha)$].



$$C^2 = A^2 + B^2 + 2AB\cos 45$$

$$\cancel{2}CC' = \cancel{2}AA' + \cancel{2}BB' + \cancel{2}\cos 45(AB' + A'B)$$

$$C' = \frac{AA' + BB' + \cos 45(AB' + A'B)}{C}$$

$$\boxed{C' = \frac{(20)(1) + (40)(2) + \cos 45[(20)(2) + (1)(40)]}{\sqrt{2000 + 800\sqrt{2}}}\ m/s}$$

At 20 sec

$A = 20\,m$

$B = 2(20) = 40\,m$

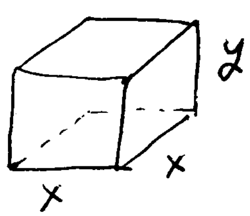
$$C^2 = 20^2 + 40^2 + 2(20)(40)\cos 45$$

$$C = \sqrt{2000 + 1600\cos 45}$$

$$= \sqrt{2000 + 800\sqrt{2}}$$

Answer 5.) ____________________

[13] 6. A box with a square base and open top must have volume of $1000cm^3$. Find the dimensions of the box that minimizes the amount of material used.



$$V = 1000 = x^2 y \Rightarrow y = \frac{1000}{x^2}$$

$$A = x^2 + 4xy = x^2 + 4x\left(\frac{1000}{x^2}\right)$$

$$y = \frac{1000}{(2000)^{2/3}}$$

$$A(x) = x^2 + \frac{4000}{x}$$

$$A'(x) = 2x - \frac{4000}{x^2} = \frac{2x^3 - 4000}{x^2}$$

$A'(x)$ DNE $\Rightarrow x = 0$

$$A'(x) = 0 : \frac{2x^3 - 4000}{x^2} = 0 \Rightarrow 2x^3 = 4000$$

$$x^3 = 2000$$

$$x = \sqrt[3]{2000}$$

Answer 6.) $\sqrt[3]{2000}\,cm \times \sqrt[3]{2000}\,cm \times \frac{1000}{(2000)^{2/3}}\,cm$

6.) Find the following for $f(x) = \frac{4-x^2}{x^2-9} = \frac{(2-x)(2+x)}{(x-3)(x+3)}$ (if they exist; if they don't exist, state so). Use this information to graph f.

Note $f'(x) = \frac{10x}{(x^2-9)^2}$ and $f''(x) = \frac{-30(x^2+3)}{(x^2-9)^3}$

[1.5] 6a.) critical numbers: 0

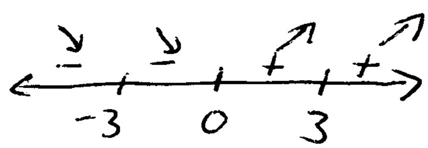
[1.5] 6b.) local maximum(s) occur at $x =$ none

[1.5] 6c.) local minimum(s) occur at $x =$ 0

[1.5] 6d.) The global maximum of f on the interval [0, 3] is none and occurs at $x =$ —

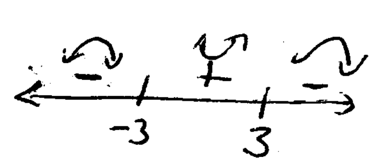
[1.5] 6e.) The global minimum of f on the interval [0, 3] is $-4/9$ and occurs at $x =$ 0

[1.5] 6f.) Inflection point(s) occur at $x =$ none



[1.5] 6g.) f increasing on the intervals $(0,3) \cup (3, \infty)$

[1.5] 6h.) f decreasing on the intervals $(-\infty, -3) \cup (-3, 0)$



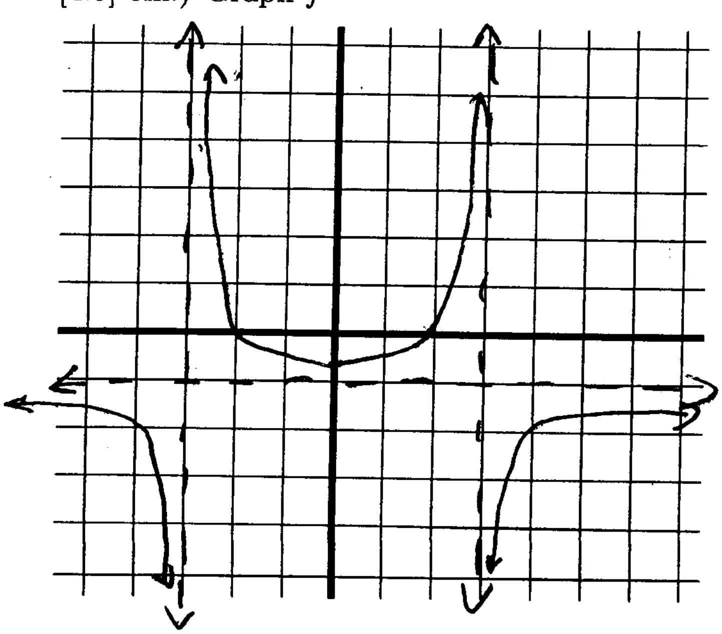
[1.5] 6i.) f is concave up on the intervals $(-3, 3)$

[1.5] 6j.) f is concave down on the intervals $(-\infty, -3) \cup (3, \infty)$

[1.5] 6k.) Equation(s) of vertical asymptote(s) $x = 3, \quad x = -3$

[4] 6l.) Equation(s) of horizontal and/or slant asymptote(s) $y = -1$

[4.5] 6m.) Graph f



$$\lim_{x \to \pm\infty} \frac{4-x^2}{x^2-9} \underset{l'H}{=} \lim_{x \to \pm\infty} \frac{-2x}{2x} = \lim_{x \to \pm\infty} (-1) = -1$$

x	y
0	$-4/9$
2	0
-2	0
4	$\frac{4-16}{16-9} = \frac{-12}{7}$
-4	$-\frac{12}{7}$