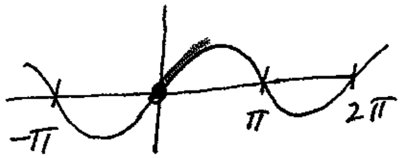
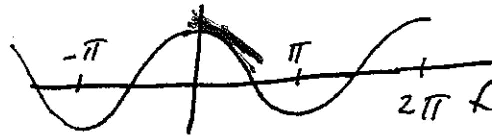
$$\sin\left(x+\frac{\pi}{2}\right) = \cos(x)$$

$f(-x) = -f(x)$

odd





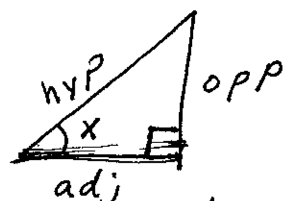
$f(-x) = f(x)$

even

$$sin(-x) = -sin(x) \qquad cos(-x) = cos(x)$$

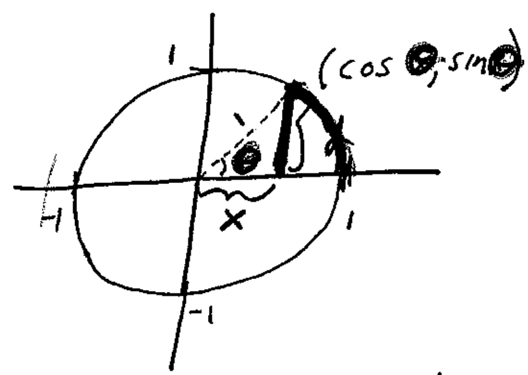
$$\frac{d}{dx}[sin(x)] = cos(x) \qquad \frac{d}{dx}[cos(x)] = -sin(x)$$

$$sin(x) = \frac{opp}{hyp} \qquad cos(x) = \frac{adj}{hyp}$$



soh cah toa

$$cos^2(x) + sin^2(x) = 1$$



$sin(x) \leq x$ for positive x

For small x, $sin(x) \sim x$

$$sin(a+b) = sin(a)cos(b) + cos(a)sin(b)$$

$$\cos(\theta) = \frac{x}{1}$$

$$cos(a+b) = cos(a)cos(b) - sin(a)sin(b)$$

$$\sin(\theta) = \frac{y}{1}$$

$g'(0)$ when $g(x) = \sin(x)$

$\shortparallel$

$$lim_{h\to 0}\frac{sin(h)}{h} = 1$$

$\frac{"0"}{0} \Rightarrow$ simplify

$$0 = lim_{h\to 0}\frac{cos(h)-1}{h} = lim_{h\to 0}\frac{cos(h)-1}{h} \cdot \frac{\cos(h)+1}{\cos(h)+1}$$

$\shortparallel$

$f'(0)$ when $f(x) = \cos(x)$

$$= \lim_{h\to 0} \frac{\cos^2(h) - 1}{h\cdot(\cos(h)+1)}$$

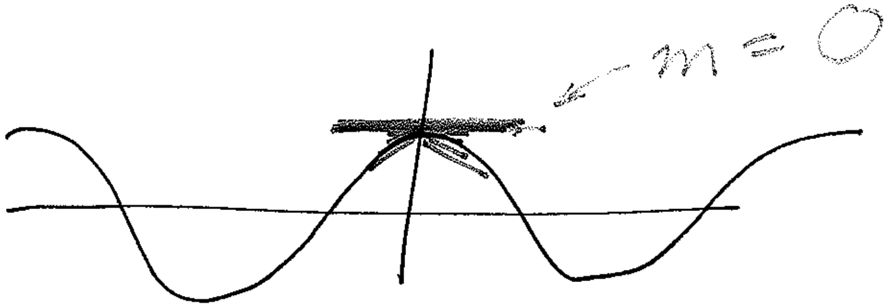
$$= \lim_{h \to 0} \frac{\cancel{1} - \sin^2(h) - \cancel{1}}{h\,(\cos(h) + 1)}$$

$$= \lim_{h \to 0} \frac{-\sin^2(h)}{h\,(\cos(h) + 1}$$

$$= \lim_{h \to 0} \frac{\sin(h)}{h} \frac{[-\sin h]}{[\cos(h) + 1]}$$

$$= \lim_{h \to 0} \frac{\sin(h)}{h} \cdot \lim_{h \to 0} \left(\frac{-\sin h}{\cos(h) + 1} \right)$$

$$= 1 \cdot \frac{0}{2} = 0$$



$$\lim_{h \to 0} \frac{\cos(h) - 1}{h}$$

$$= \lim_{h \to 0} \frac{\cos(0+h) - \cos(0)}{h}$$

$= f'(0)$ where $f(x) = \cos(x)$

$= 0$ graphically

Let $f(x) = cos(x)$.

Then $f'(x) = \lim_{h \to 0} \frac{\cos(x+h) - \cos(x)}{h}$

minus

$$= \lim_{h \to 0} \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h}$$

$$= \lim_{h \to 0} \left[\cos(x) \frac{[\cos(h) - 1]}{h} - \sin x \cdot \frac{\sin h}{h} \right]$$

$$= \lim_{h \to 0} \cos(x) \left[\frac{\cos(h) - 1}{h} \right] - \lim_{h \to 0} \sin x \cdot \frac{\sin h}{h}$$

$$= \cos(x) \underbrace{\lim_{h \to 0} \frac{\cos(h) - 1}{h}}_{\to 0} - \sin(x) \underbrace{\lim_{h \to 0} \frac{\sin h}{h}}_{\to 1}$$

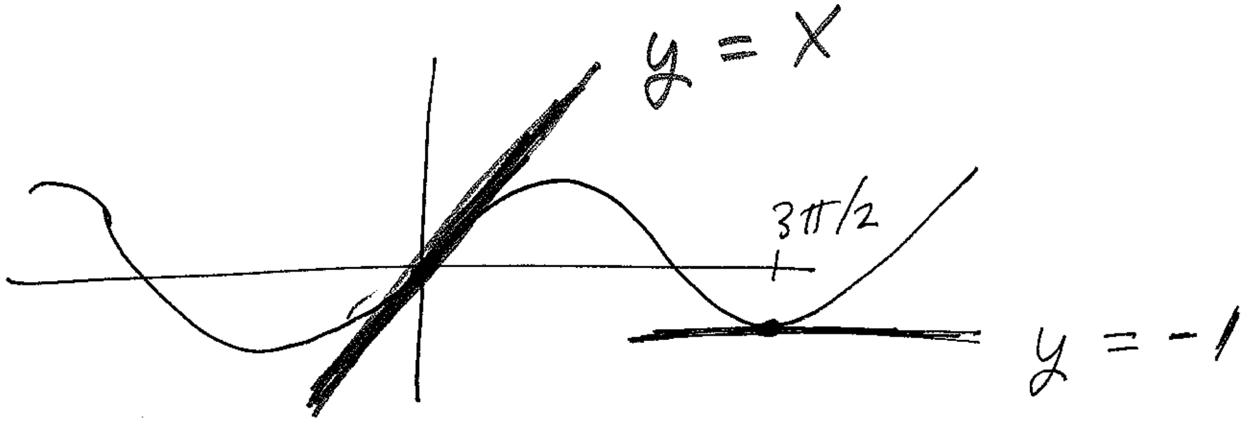
$$= -\sin(x)$$

Similarly $\frac{d}{dx}[sin(x)] = cos(x)$

Find equation of tangent line to $y = sin(x)$ at $\underline{x = 0}$

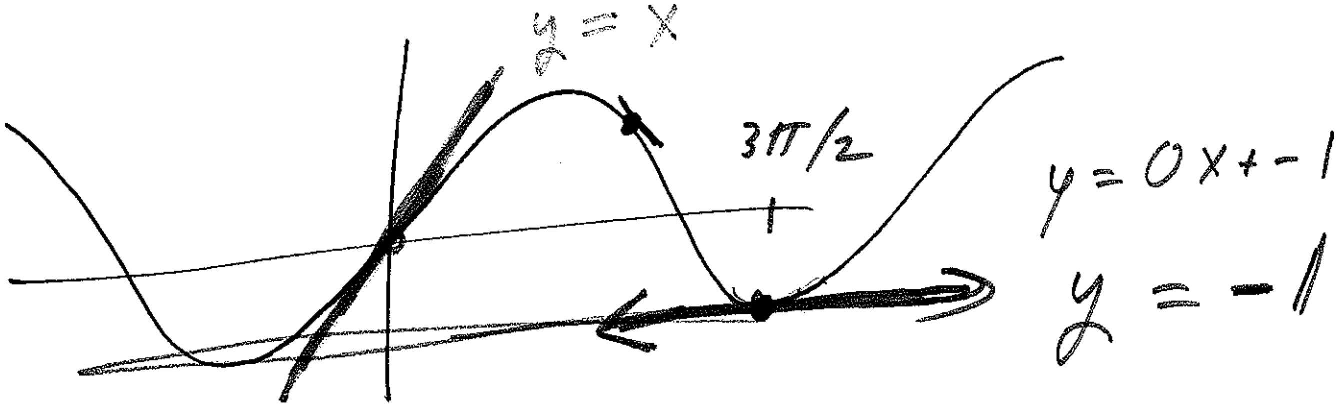
$$\text{slope} = \frac{d}{dx}[\sin(x)]_{x=0} = \cos(0) = 1$$

$$\frac{y - y_1}{x - x_1} = 1 = \frac{y - \sin(0)}{x - 0} = \frac{y}{x} \Rightarrow y = x$$



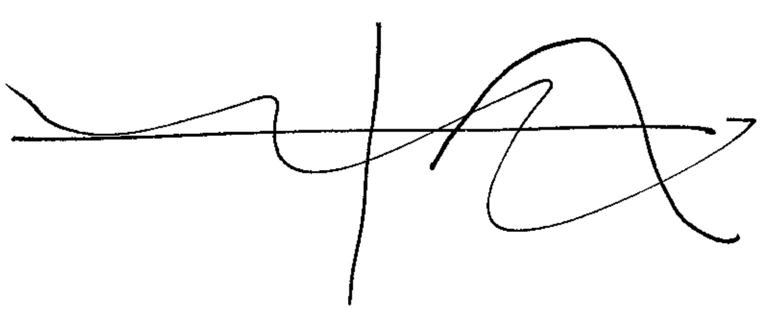
For small x $\sin x \sim x$

For x ~~at~~ near $\frac{3\pi}{2}$ $\sin x \sim -1$



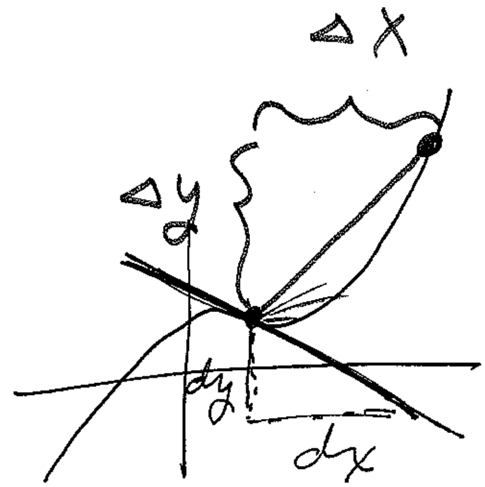
$\sin x \sim x$ for x near 0

$\sin x \sim -1$ for x near $\frac{3\pi}{2}$



$z = (f \circ g)(x)$ $\quad (f \circ g)'(x) = \frac{dz}{dx}$

$\frac{\Delta y}{\Delta x}$ = slope of secant line



$$x \to y = g(x) \to z = f(y) = f(g(x))$$

$y = g(x)$ $\quad f(y) = z$ $\quad f'(y) = \frac{dz}{dy}$

$g'(x) = dy/dx$

Chain rule: $[f(g(x))]' = f'(g(x)) \cdot g'(x)$

$$\frac{dz}{dx} = \frac{dz}{\cancel{dy}}\frac{\cancel{dy}}{dx}$$

$$lim_{\Delta x \to 0}\frac{\Delta z}{\Delta x} = lim_{\Delta x \to 0}\frac{\Delta z}{\cancel{\Delta y}}\frac{\cancel{\Delta y}}{\Delta x}$$

$\frac{dy}{dx}$ = slope of tangent line

Examples:

$[\underline{cos}(\boxed{e^x + 5x^2})]' =$

$= [-\sin(e^x + 5x^2)] \cdot (e^x + 5x^2)'$

$= [-\sin(e^x + 5x^2)](e^x + 10x)$

$$\left(x^3\cdot[\sin(x^2)]\right)\cdot[\cos(x^{-1}+5)])' =$$

$$= \left(x^3\cdot\sin(x^2)\right)'\cos(x^{-1}+5) + x^3\sin(x^2)\frac{d[\cos(x^{-1}+5)]}{dx}$$

$$= \left[3x^2\cdot\sin(x^2) + x^3(\sin(x^2))'\right]\cos(x^{-1}+5)$$

$$+ x^3\sin(x^2)\cdot\left(-\sin(x^{-1}+5)\right)(x^{-1}+5)'$$

$$= \left[3x^2\sin(x^2) + x^3\cos(x^2)\cdot 2x\right]\cos(x^{-1}+5)$$

$$+ x^3\sin(x^2)\cdot\left(-\sin(x^{-1}+5)\right)(-x^{-2})$$

$$(3x^2\cdot\sin(x^2) + x^3\cos(x^2)2x)\cos(x^{-1}+5)$$

$$+ x^3\sin(x^2)\cdot\left(-\sin(x^{-1}+5)\right)(-x^{-2})$$

$$\left[\sqrt{\frac{sin(x^{-1})}{e^{x^2}}}\right]' = \left[\left(\boxed{\frac{\sin(x^{-1})}{e^{x^2}}}\right)^{1/2}\right]'$$

$$= \left[\left(e^{-x^2} \cdot \sin(x^{-1})\right)^{1/2}\right]'$$

$$= \frac{1}{2}\left(\underline{e^{-x^2} \cdot \sin(x^{-1})}\right)^{-1/2} \cdot \left[e^{-x^2} \cdot (-2x)\sin(x^{-1}) + e^{-x^2} \cdot \cos(x^{-1})(-x^{-2})\right]$$

$$= \frac{1}{2\left(e^{-x^2} \cdot \sin(x^{-1})\right)^{1/2}}\left[-2xe^{-x^2}\sin(x^{-1}) + e^{-x^2}(-x^{-2})(\cos(x^{-1}))\right]$$

$$\left[\left(e^{-x^2}\cdot\sin(x^{-1})\right)^{1/2}\right]'$$

$$=\frac{1}{2}\left(e^{-x^2}\cdot\sin(x^{-1})\right)^{1/2}\left(e^{-x^2}\cdot\sin(x^{-1})\right)'$$

$$=\frac{1}{2}\left(e^{-x^2}\cdot\sin(x^{-1})\right)^{1/2}\left(e^{-x^2}(-x^2)'\sin(x^{-1}) + e^{-x^2}\cos(x^{-1})\frac{d}{dx}(x^{-1})\right)$$

$$= \text{etc}$$

$$[e^{sin(e^{x^2+3x})}]' =$$

$$= e^{\sin(e^{x^2+3x})} \cdot \cos(e^{x^2+3x}) \cdot e^{x^2+3x} \cdot (2x+3)$$

$$e^{\sin(e^{x^2+3x})} \cdot (\sin(e^{x}+3x))'$$