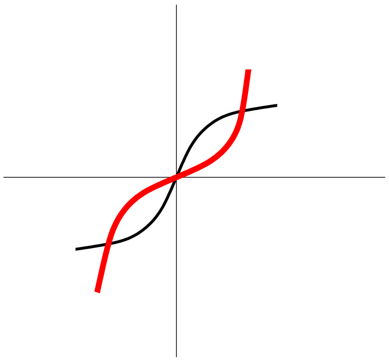
Find the area bounded by the functions $y = 2x^3$ and $y = 2x^{\frac{1}{3}}$.

1.) Find points of intersection:

$2x^3 = 2x^{\frac{1}{3}}$ implies $x^3 = x^{\frac{1}{3}}$ implies $x^9 = x$. Thus $x^9 - x = x(x^8 - 1) = 0$.
Hence $x = 0$ and $x^8 - 1 = 0$. $x^8 = 1$ implies $x = 1, -1$

Hence the functions $y = 2x^3$ and $y = 2x^{\frac{1}{3}}$ intersect when $x = -1, 0, 1$

2.) Draw a rough graph:

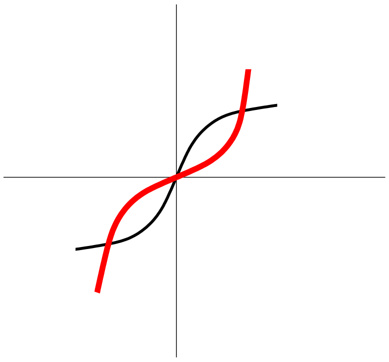


3.) Find area:

Method 1: Use vertical rectangles:

$\int_{-1}^{0}[2x^3 - 2x^{\frac{1}{3}}]dx + \int_0^1[2x^{\frac{1}{3}} - 2x^3]dx$

Method 2: Use horizontal rectangles:

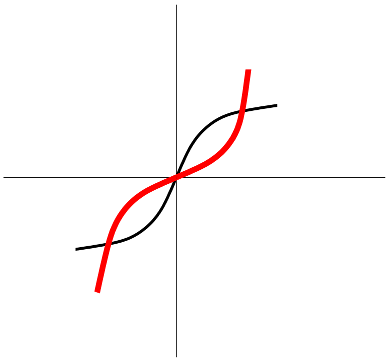


If $y = 2x^3$, then $x = (\frac{y}{2})^{\frac{1}{3}}$. If $y = 2x^{\frac{1}{3}}$, then $x = (\frac{y}{2})^3$.

When $x = -1, y = -2$. When $x = 0, y = 0$. When $x = 1, y = 2$.

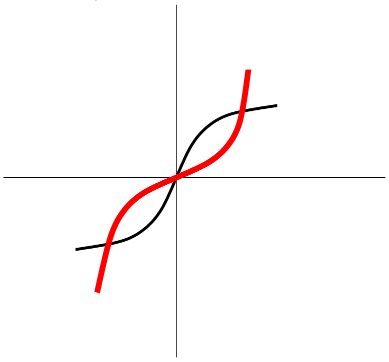
$\int_{-2}^{0}[(\frac{y}{2})^3 - (\frac{y}{2})^{\frac{1}{3}}]dy + \int_0^2[(\frac{y}{2})^{\frac{1}{3}} - (\frac{y}{2})^3]dy$

Find the volume of the object obtained by rotating the area bounded by the functions $y = 2x^3$ and $y = 2x^{\frac{1}{3}}$ about the $x-$axis $(y = 0)$:



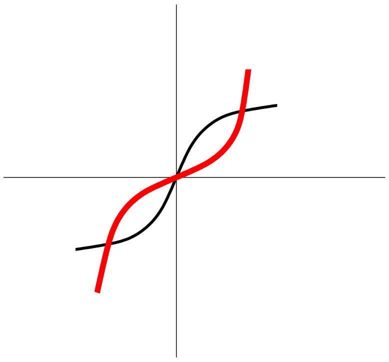
$$\pi \int_{-1}^{0} [(2x^{\frac{1}{3}})^2 - (2x^3)^2]dx + \pi \int_{0}^{1} [(2x^{\frac{1}{3}})^2 - (2x^3)^2]dx = \int_{-1}^{1} [(2x^{\frac{1}{3}})^2 - (2x^3)^2]dx$$

Find the volume of the object obtained by rotating the area bounded by the functions $y = 2x^3$ and $y = 2x^{\frac{1}{3}}$. about $y = 4$:



$$\pi \int_{-1}^{0} [(4 - 2x^{\frac{1}{3}})^2 - (4 - 2x^3)^2]dx + \pi \int_{0}^{1} [(4 - 2x^3)^2 - (4 - 2x^{\frac{1}{3}})^2]dx$$

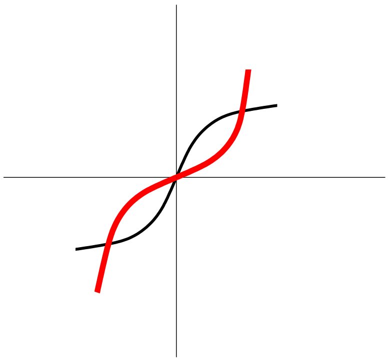
Find the volume of the object obtained by rotating the area bounded by the functions $y = 2x^3$ and $y = 2x^{\frac{1}{3}}$. about $y = -2$:



$$\pi \int_{-1}^{0} [(2x^3 - (-2))^2 - (2x^{\frac{1}{3}} - (-2))^2]dx + \pi \int_{0}^{1} [(2x^{\frac{1}{3}} - (-2))^2 - (2x^3 - (-2))^2]dx$$

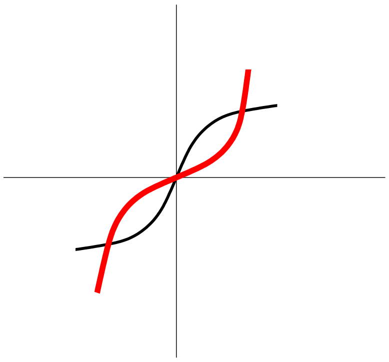
$$= \pi \int_{-1}^{0} [(2x^3 + 2)^2 - (2x^{\frac{1}{3}} + 2)^2]dx + \pi \int_{0}^{1} [(2x^{\frac{1}{3}} + 2)^2 - (2x^3 + 2)^2]dx$$

Find the volume of the object obtained by rotating the area bounded by the functions $y = 2x^3$ $(x = (\frac{y}{2})^{\frac{1}{3}})$and $y = 2x^{\frac{1}{3}}$ $(x = (\frac{y}{2})^3)$ about the $y-$axis $(x = 0)$:



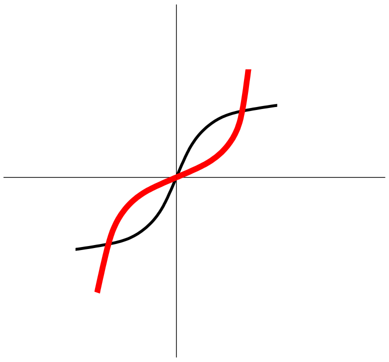
$$\pi \int_{-2}^{0}[((\tfrac{y}{2})^{\frac{1}{3}})^2 - ((\tfrac{y}{2})^3)^2]dy + \pi \int_{0}^{2}[((\tfrac{y}{2})^{\frac{1}{3}})^2 - ((\tfrac{y}{2})^3)^2]dy = \pi \int_{-2}^{2}[((\tfrac{y}{2})^{\frac{1}{3}})^2 - ((\tfrac{y}{2})^3)^2]dy$$

Find the volume of the object obtained by rotating the area bounded by the functions $y = 2x^3$ $(x = (\frac{y}{2})^{\frac{1}{3}})$and $y = 2x^{\frac{1}{3}}$ $(x = (\frac{y}{2})^3)$ about $x = 8$:



$$\pi \int_{-2}^{0}[(8 - (\tfrac{y}{2})^{\frac{1}{3}})^2 - (8 - (\tfrac{y}{2})^3)^2]dy + \pi \int_{0}^{2}[(8 - (\tfrac{y}{2})^3)^2 - (8 - (\tfrac{y}{2})^{\frac{1}{3}})^2]dy$$

Find the volume of the object obtained by rotating the area bounded by the functions $y = 2x^3$ $(x = (\frac{y}{2})^{\frac{1}{3}})$and $y = 2x^{\frac{1}{3}}$ $(x = (\frac{y}{2})^3)$ about $x = -1$:



$$\pi \int_{-2}^{0}[((\tfrac{y}{2})^3 - 1)^2 - ((\tfrac{y}{2})^{\frac{1}{3}} - 1)^2]dy + \pi \int_{0}^{2}[((\tfrac{y}{2})^{\frac{1}{3}} - 1)^2 - ((\tfrac{y}{2})^3 - 1)^2]dy$$