Suppose f integrable
(Note f continuous implies f integrable).

If n equal subdivisions: $\Delta x = \frac{b-a}{n}$ and if we use right-hand endpoints: $x_i = a + i\Delta x = a + \frac{(b-a)i}{n}$

$$\int_a^b f(x)dx = lim_{n\to\infty}\Sigma_{i=1}^n f(a + \frac{(b-a)i}{n})(\frac{b-a}{n})$$

Evaluate the limit by recognizing the sum as a Riemann sum for a function defined on [0, 1]

1.) $lim_{n\to\infty}\Sigma_{i=1}^n sin(\frac{i}{n})\frac{1}{n}$

2.) $lim_{n\to\infty}\Sigma_{i=1}^n \frac{i^5}{n^6}$

The Fundamental Theorem of Calculus: Suppose f continuous on $[a, b]$.

1.) If $G(x) = \int_a^x f(t)dt$, then $G'(x) = f(x)$.

2.) $\int_a^b f(t)dt = F(b) - F(a)$ where F is any antiderivative of f, that is $F' = f$.

The Fundamental Theorem of Calculus: Suppose f continuous on $[a, b]$.

1.) If $\frac{d}{dx}[\int_a^x f(t)dt] = f(x)$.

2.) $\int_a^b F'(t)dt = F(b) - F(a)$.

Examples:

1.) If $G_1(x) = \int_0^x t^2 dt$, then $G_1'(x) =$ ________________.

2.) If $G_2(x) = \int_5^x t^2 dt$, then $G_2'(x) =$ ________________.

3.) If $G_3(x) = \int_{-2}^x sin(t^2)dt$, then $G_3'(x) =$ ________________.

4.) If $G_4(x) = \int_4^x tan(\frac{t^3}{t+1})dt$, then $G_4'(x) =$ ________________.

5.) If $G_5(x) = \int_1^x \sqrt{3t-5}dt$, then $G_5'(x) =$ ________________.

Examples:

1.) If $H_1(x) = \int_0^{x^3} t^2 dt$, then $H_1'(x) =$ ________________.

2.) If $H_2(x) = \int_5^{x^3} t^2 dt$, then $H_2'(x) =$ ________________.

3.) If $G(x) = \int_2^{ln(x)} \frac{t}{t+1} dt$, then $G'(x) =$ ________________.

4.) If $G(x) = \int_3^{\frac{1}{x}} sec(t)dt$, then $G'(x) =$ ________________.

The Fundamental Theorem of Calculus: Suppose f continuous on $[a, b]$.

1.) If $G(x) = \int_a^x f(t)dt$, then $G'(x) = f(x)$.

Proof

$$G'(x) = lim_{h\to 0}\frac{G(x+h)-G(x)}{h},$$

$$= lim_{h\to 0}\frac{\int_a^{x+h} f(t)dt - \int_a^x f(t)dt}{h}$$

$$= lim_{h\to 0}\frac{\int_x^{x+h} f(t)dt}{h}$$

$$\leq lim_{h\to 0}\frac{\int_x^{x+h} M_h dt}{h}$$

where $M_h = max\{f(t) \mid x \leq t \leq t+h\}$

(Note M_h exists by extreme value thm)

$$\leq lim_{h\to 0}\frac{(M_h)(h)}{h}$$

$$\leq lim_{h\to 0}M_h = f(x)$$

Similarly $G'(x) \geq f(x)$

(using $m_h = min\{f(t) \mid x \leq t \leq t+h\}$

The Fundamental Theorem of Calculus: Suppose f continuous on $[a, b]$.

2.) $\int_a^b f(t)dt = F(b) - F(a)$ where F is any antiderivative of f, that is $F' = f$.

Proof

Let $G(x) = \int_a^x f(t)dt$. Then $G'(x) = f(x)$ (ie, G is an antiderivative of f).

Let F be any antiderivative of f.

Then $F(x) = G(x) + C = \int_a^x f(t)dt + C$ for some constant C.

Thus $F(b) - F(a) = G(b) + C - [G(a) + C]$

$= G(b) - G(a) = \int_a^b f(t)dt - \int_a^a f(t)dt = \int_a^b f(t)dt$.

$$\int_0^\pi cos^3(x)dx = \int_0^\pi cos^2(x)cos(x)dx = \int_0^\pi (1-sin^2(x))cos(x)dx$$
$$= \int_0^0 (1 - u^2)du = 0$$

Let $u = sin(x)$, $du = cos(x)dx$,
when $x = 0$, $u = sin(0) = 0$, when $x = \pi$, $u = sin(\pi) = 0$

Shortcut method: Use symmetry.

For example:
If f is an odd function ($f(-x) = -f(x)$), then $\int_{-a}^a f(x)dx = 0$
If f is an even function, then $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$