Use the Riemann sum definition of integral to evaluate $\int_2^5 x^3 dx$

$\Delta x = \frac{5-2}{n} = \frac{3}{n}$, $x_i = 2 + \frac{3i}{n}$, $f(x) = x^3$.

$$\int_2^5 x^3 dx = lim_{n\to\infty} \Sigma_{i=1}^n f(x_i)\Delta x = lim_{n\to\infty} \Sigma_{i=1}^n (2 + \frac{3i}{n})^3 (\frac{3}{n})$$

$$= lim_{n\to\infty} \Sigma_{i=1}^n [2^3 + 3(2^2)(\frac{3i}{n}) + 3(2)(\frac{3i}{n})^2 + (\frac{3i}{n})^3](\frac{3}{n})$$

$$lim_{n\to\infty} \Sigma_{i=1}^n [8 + \frac{36i}{n} + \frac{54i^2}{n^2} + \frac{27i^3}{n^3}](\frac{3}{n})$$

$$lim_{n\to\infty} \Sigma_{i=1}^n [\frac{24}{n} + \frac{108i}{n^2} + \frac{162i^2}{n^3} + \frac{81i^3}{n^4}]$$

$$lim_{n\to\infty} [\Sigma_{i=1}^n \frac{24}{n} + \Sigma_{i=1}^n \frac{108i}{n^2} + \Sigma_{i=1}^n \frac{162i^2}{n^3} + \Sigma_{i=1}^n \frac{81i^3}{n^4}]$$

$$lim_{n\to\infty} [\frac{24}{n} \Sigma_{i=1}^n 1 + \frac{108}{n^2} \Sigma_{i=1}^n i + \frac{162}{n^3} \Sigma_{i=1}^n i^2 + \frac{81}{n^4} \Sigma_{i=1}^n i^3]$$

$$lim_{n\to\infty} [24 + \frac{108}{n^2}(\frac{n(n+1)}{2}) + \frac{162}{n^3}(\frac{n(n+1)(2n+1)}{6}) + \frac{81}{n^4}(\frac{n(n+1)}{2})^2]$$

$$lim_{n\to\infty} [24 + \frac{108}{n}(\frac{n+1}{2}) + \frac{162}{n^2}(\frac{2n^2+3n+1}{6}) + \frac{81}{n^2}(\frac{n^2+2n+1}{4})]$$

$$lim_{n\to\infty} [24 + \frac{108}{n}(\frac{n(1+\frac{1}{n})}{2}) + \frac{162}{n^2}(\frac{n^2(2+\frac{3}{n}+\frac{1}{n^2})}{6}) + \frac{81}{n^2}(\frac{n^2(1+\frac{2}{n}+\frac{1}{n^2})}{4})]$$

$$lim_{n\to\infty} [24 + 54(1 + \frac{1}{n}) + 27((2 + \frac{3}{n} + \frac{1}{n^2})) + 81(\frac{(1+\frac{2}{n}+\frac{1}{n^2})}{4})]$$

$$= [24 + 54(1) + 27(2) + 81(\frac{1}{4})] = [\frac{24(4)+54(4)+27(8)+81}{4}]$$

$$= \frac{96+216+216+81}{4} = \frac{609}{4}$$

Check: $\int_2^5 x^3 dx = \frac{x^4}{4}|_2^5 = \frac{5^4-2^4}{4} = \frac{625-16}{4} = \frac{609}{4}$

$\Sigma_{i=1}^n i = \frac{n(n+1)}{2}$, $\Sigma_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$, $\Sigma_{i=1}^n i^3 = [\frac{n(n+1)}{2}]^2$