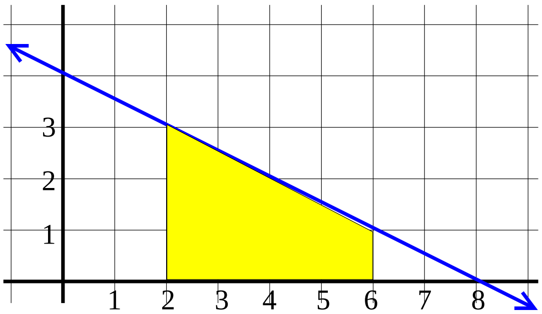
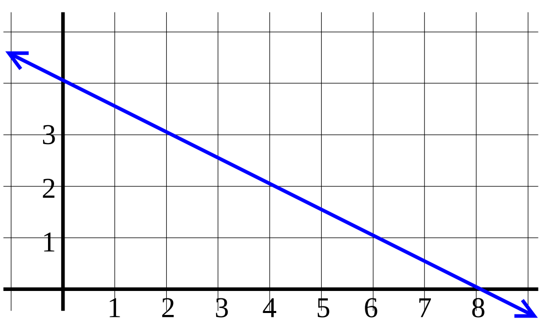
Section 5.1

Find the area under the curve $f(x) = -\frac{1}{2}x + 4$, above the $x-$axis and between $x = 2$ and $x = 6$.



Method 1: In this case our function is very simple, so we can determine the area without calculus:

Inscribed rectangles with $\Delta x = \frac{1}{2}$:



$f(\frac{5}{2})(\frac{1}{2}) + f(3)(\frac{1}{2}) + f(\frac{7}{2})(\frac{1}{2}) + f(4)(\frac{1}{2})$
$\quad + f(\frac{9}{2})(\frac{1}{2}) + f(5)(\frac{1}{2}) + f(\frac{11}{2})(\frac{1}{2}) + f(6)(\frac{1}{2})$

$= [-\frac{1}{2}(\frac{5}{2}) + 4](\frac{1}{2}) + [-\frac{1}{2}(3) + 4](\frac{1}{2}) + [-\frac{1}{2}(\frac{7}{2}) + 4](\frac{1}{2})$

$+ [-\frac{1}{2}(4) + 4](\frac{1}{2}) + [-\frac{1}{2}(\frac{9}{2}) + 4](\frac{1}{2}) + [-\frac{1}{2}(5) + 4](\frac{1}{2})$

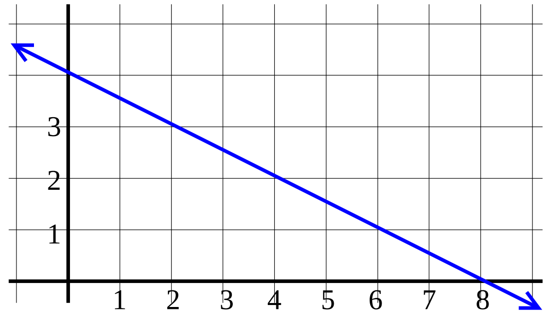
$+ [-\frac{1}{2}(\frac{11}{2}) + 4](\frac{1}{2}) + [-\frac{1}{2}(6) + 4](\frac{1}{2})$

$= \frac{11}{4}(\frac{1}{2}) + \frac{5}{2}(\frac{1}{2}) + \frac{9}{4}(\frac{1}{2}) + 2(\frac{1}{2}) + \frac{7}{4}(\frac{1}{2}) + \frac{3}{2}(\frac{1}{2}) + \frac{5}{4}(\frac{1}{2}) + 1(\frac{1}{2})$

$= \frac{15}{2}$

Method 2: Estimate using rectangles.

Inscribed rectangles with $\Delta x = 1$:

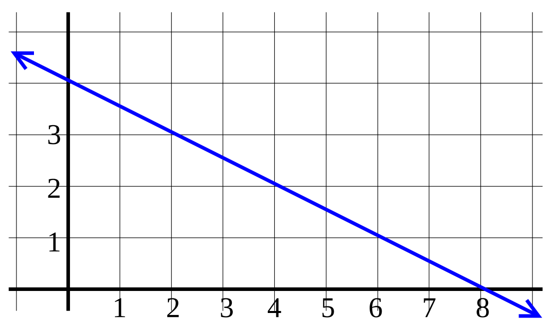


$f(3)(1) + f(4)(1) + f(5)(1) + f(6)(1) =$

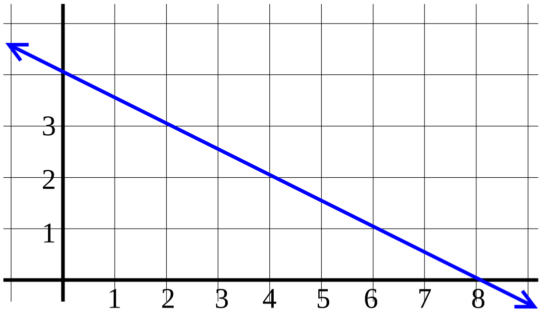
$= [-\frac{1}{2}(3) + 4](1) + [-\frac{1}{2}(4) + 4](1)$
$\quad + [-\frac{1}{2}(5) + 4](1) + [-\frac{1}{2}(6) + 4](1)$

$= \frac{5}{2}(1) + 2(1) + \frac{3}{2}(1) + 1(1) = 7$

Inscribed rectangles with $\Delta x = \frac{6-2}{n} = \frac{4}{n}$:



Circumscribed rectangles with $\Delta x = 1$:



$f(2)(1) + f(3)(1) + f(4)(1) + f(5)(1) =$

$= [-\frac{1}{2}(2) + 4](1) + [-\frac{1}{2}(3) + 4](1)$
$\quad + [-\frac{1}{2}(4) + 4](1) + [-\frac{1}{2}(5) + 4](1)$

$= 3 + \frac{5}{2}(1) + 2(1) + \frac{3}{2}(1) = 9$

Defn: $\int_a^b f(x)dx = lim_{n\to\infty}\Sigma_{i=1}^n f(x_i)\Delta x$

If f is continuous, can use inscribed rectangles, circumscribed rectangles, all left-hand endpoints, all right-hand endpoints, or all midpoints, etc.

If $\Delta x = \frac{b-a}{n}$ and if right-hand endpoints are used, then $x_i = a + i\Delta x = a + \frac{(b-a)i}{n}$

$$\int_a^b f(x)dx = lim_{n\to\infty}\Sigma_{i=1}^n f(a + \frac{(b-a)i}{n})(\frac{b-a}{n})$$

Properties of the definite integral

$\int_a^a f(x)dx = 0$

$\int_a^b f(x)dx = -\int_b^a f(x)dx$

$\int_a^b kf(x)dx = k\int_a^b f(x)dx$

$\int_a^b (f_1 + f_2)(x)dx = \int_a^b f_1(x)dx + \int_a^b f_2(x)dx$

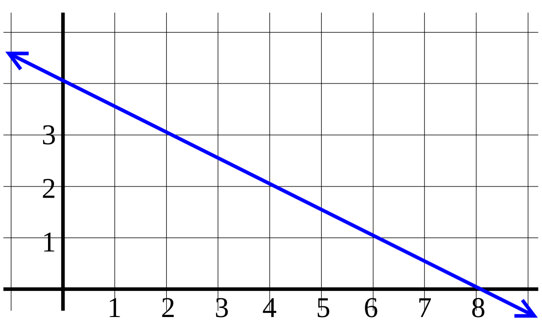
$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$

If $f_1(x) \leq f_2(x)$, then $\int_a^b f_1(x)dx \leq \int_a^b f_2(x)dx$

If $m \leq f(x) \leq M$ then $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$

Estimate the distance traveled between $t = 2$ and $t = 6$ if the velocity is given by the function $f(t) = -\frac{1}{2}t + 4$.

Estimate using inscribed rectangles with $\Delta t = 1$:

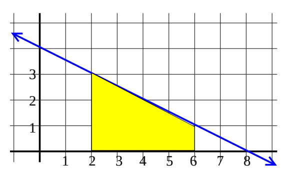


$f(3)(1) + f(4)(1) + f(5)(1) + f(6)(1) =$

$= [-\frac{1}{2}(3) + 4](1) + [-\frac{1}{2}(4) + 4](1)$
$\quad + [-\frac{1}{2}(5) + 4](1) + [-\frac{1}{2}(6) + 4](1)$

$= \frac{5}{2}(1) + 2(1) + \frac{3}{2}(1) + 1(1) = 7$

Find the distance traveled between $t = 2$ and $t = 6$ if the velocity is given by the function $f(t) = -\frac{1}{2}t + 4$.



Method 1: In this case our function is very simple, so we can determine the area without calculus:

Method 2: Use calculus by estimating with rectangles and taking limit.

$$\text{Area} = = lim_{n\to\infty}\Sigma_{i=1}^n f(a + \frac{(b-a)i}{n})(\frac{b-a}{n})$$
$$= lim_{n\to\infty}\Sigma_{i=1}^n f(2 + \frac{4i}{n})(\frac{4}{n})$$
$$= lim_{n\to\infty}\Sigma_{i=1}^n [-\frac{1}{2}(2 + \frac{4i}{n}) + 4](\frac{4}{n}) = 8$$

Method 3 (section 5.3): Use calculus by integrating.

$$\int_2^6 (-\frac{1}{2}t + 4)dt = (-\frac{1}{4}t^2 + 4t)|_2^6$$
$$= (-\frac{1}{4}(6)^2 + 4(6)) - (-\frac{1}{4}(2)^2 + 4(2))$$
$$= -9 + 24 - (-1 + 8) = 15 - 7 = 8$$